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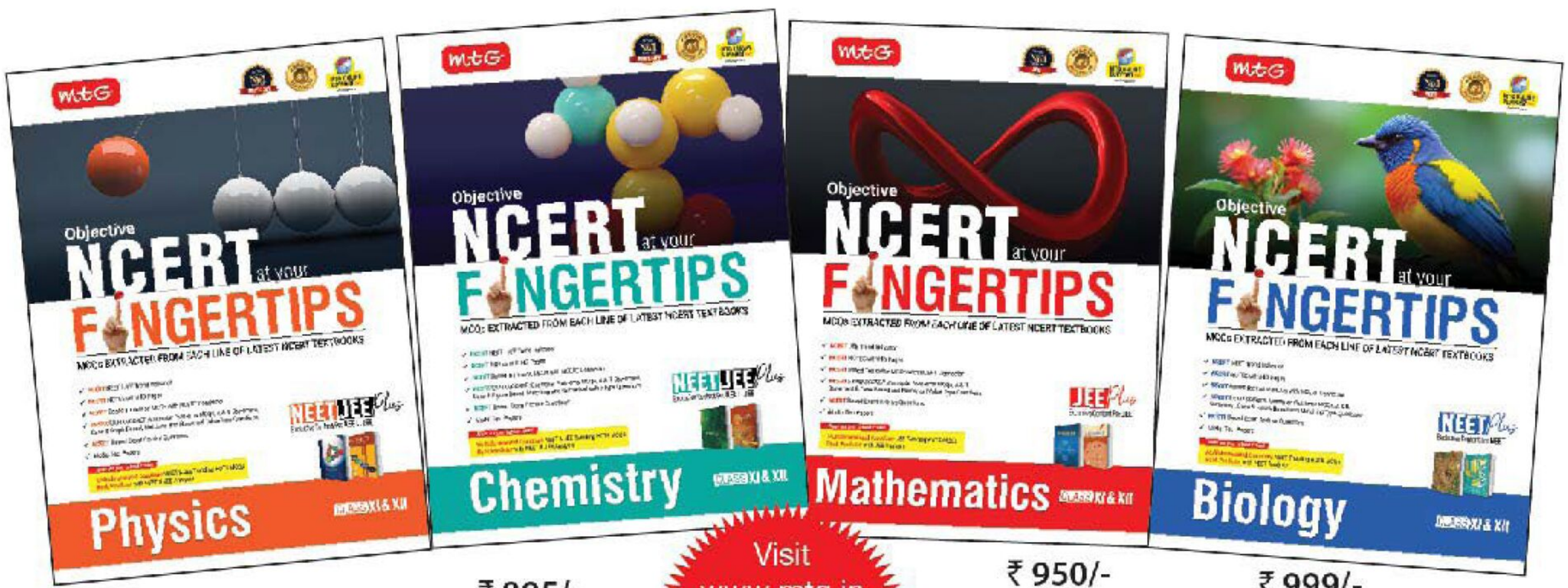


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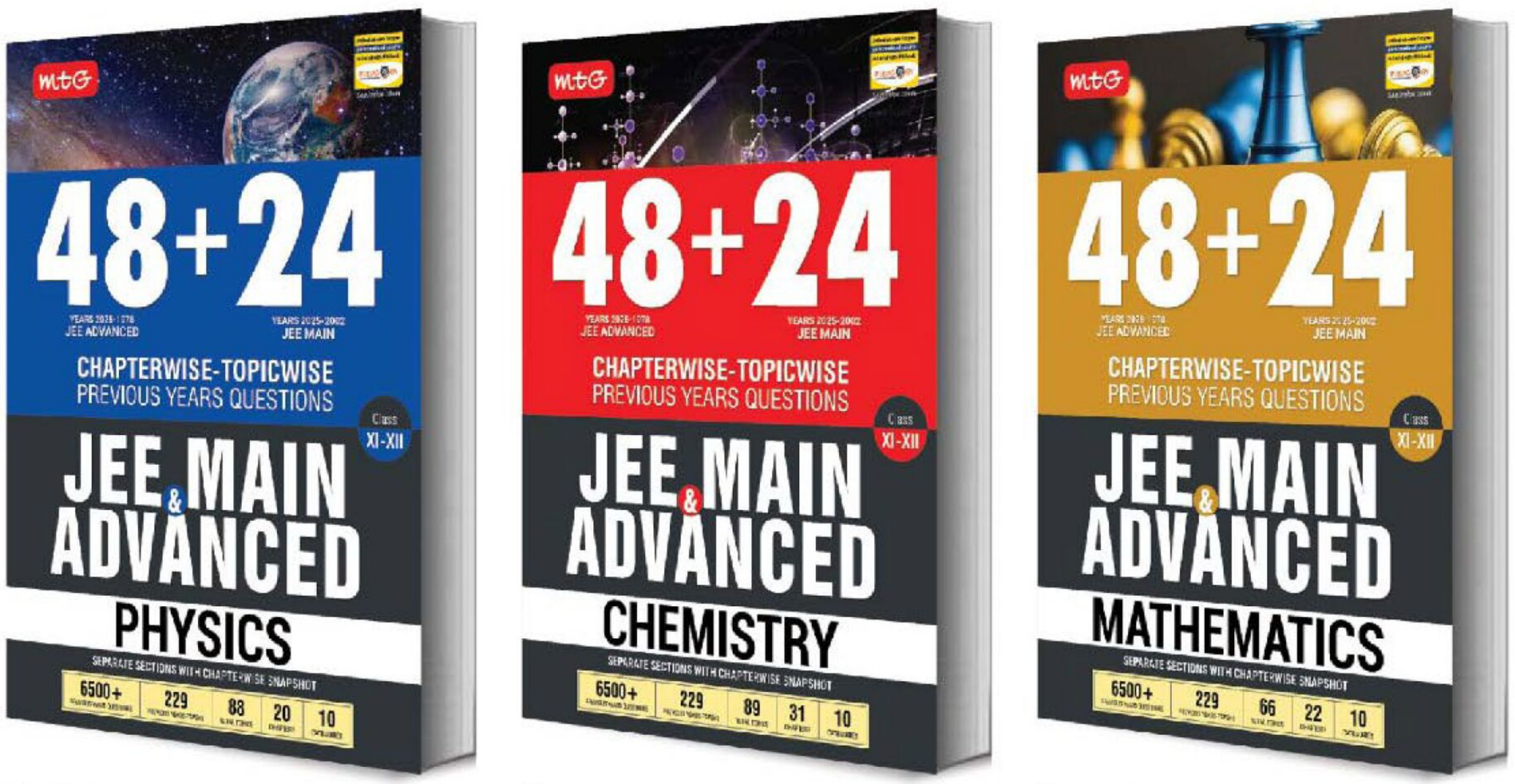


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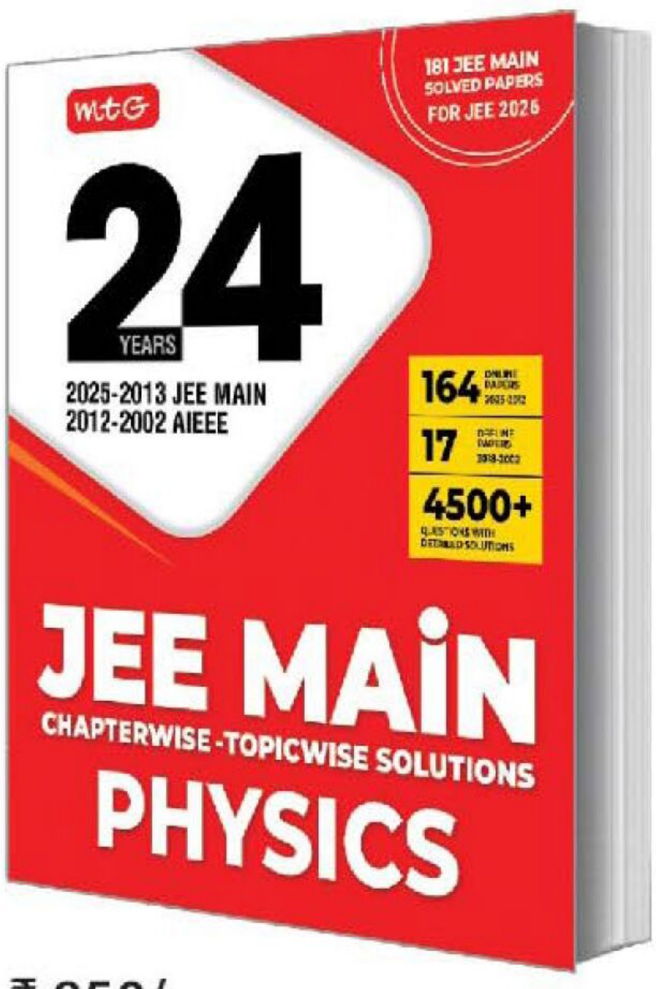


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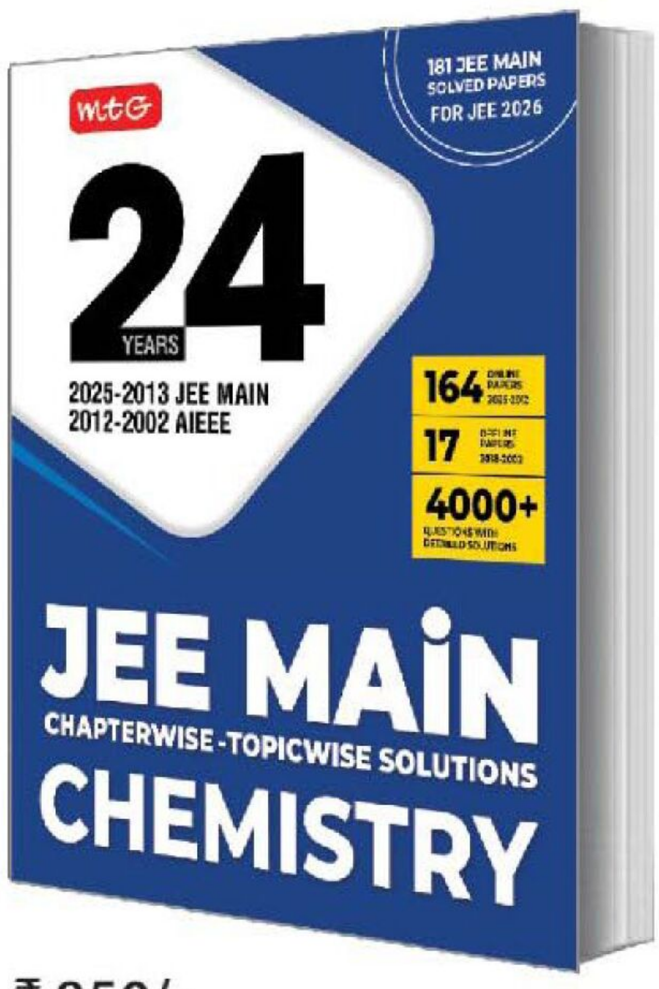


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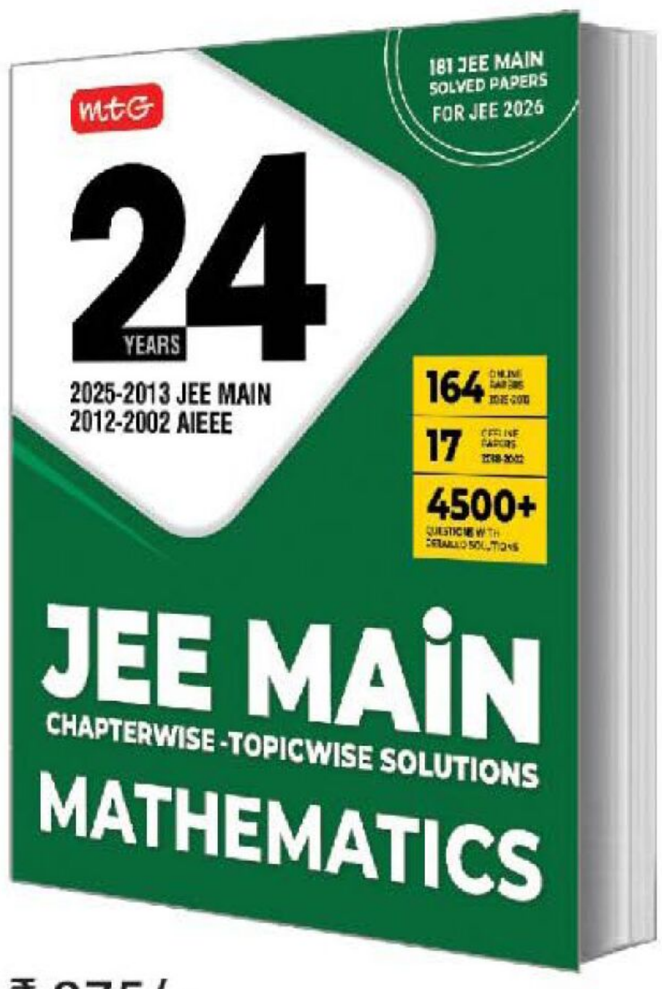


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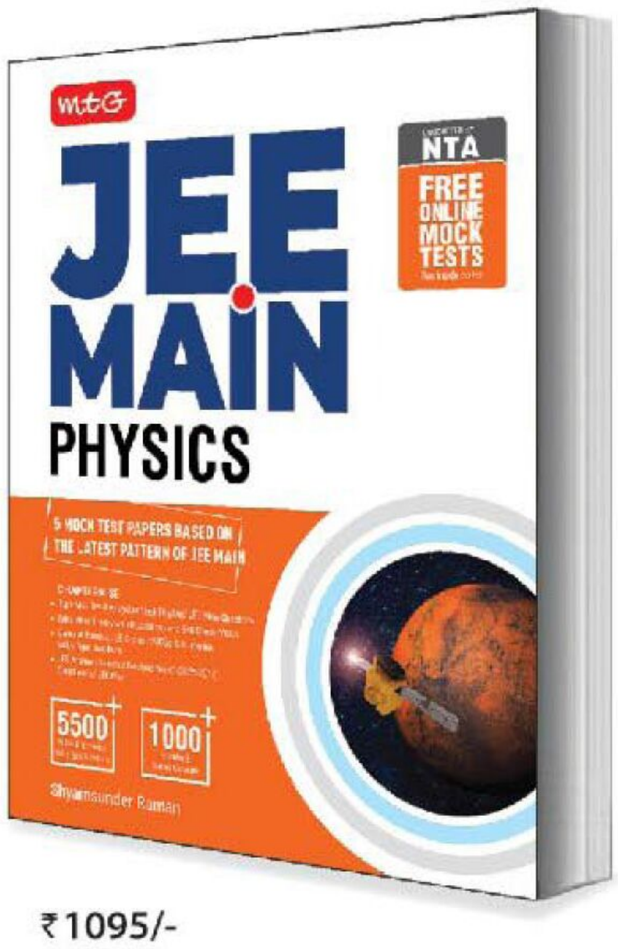


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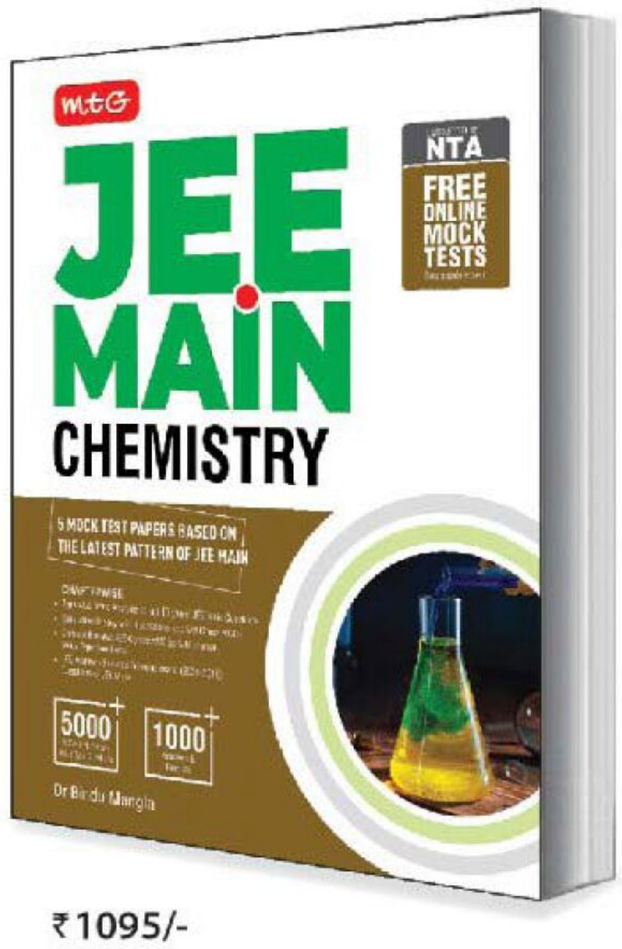


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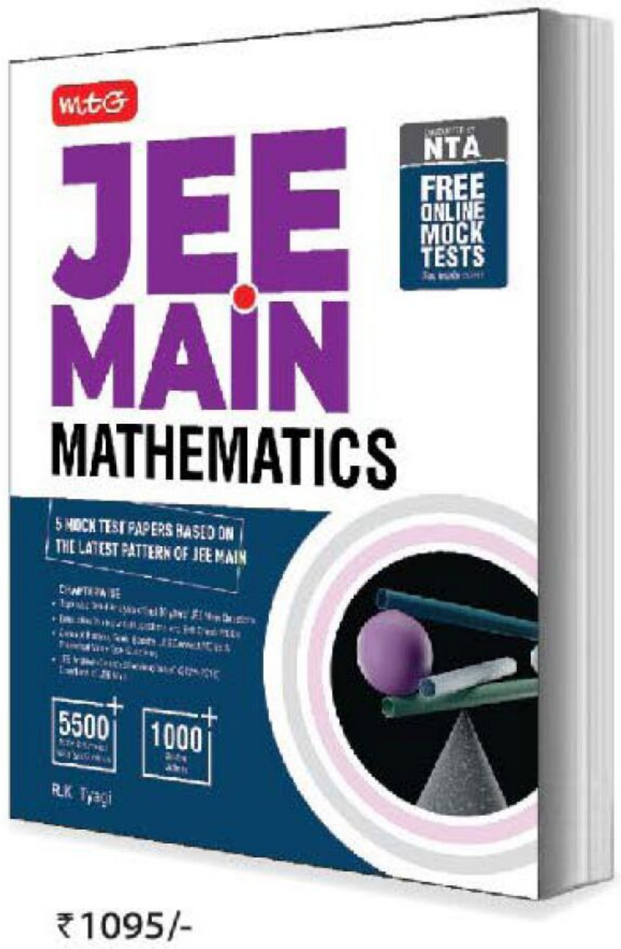


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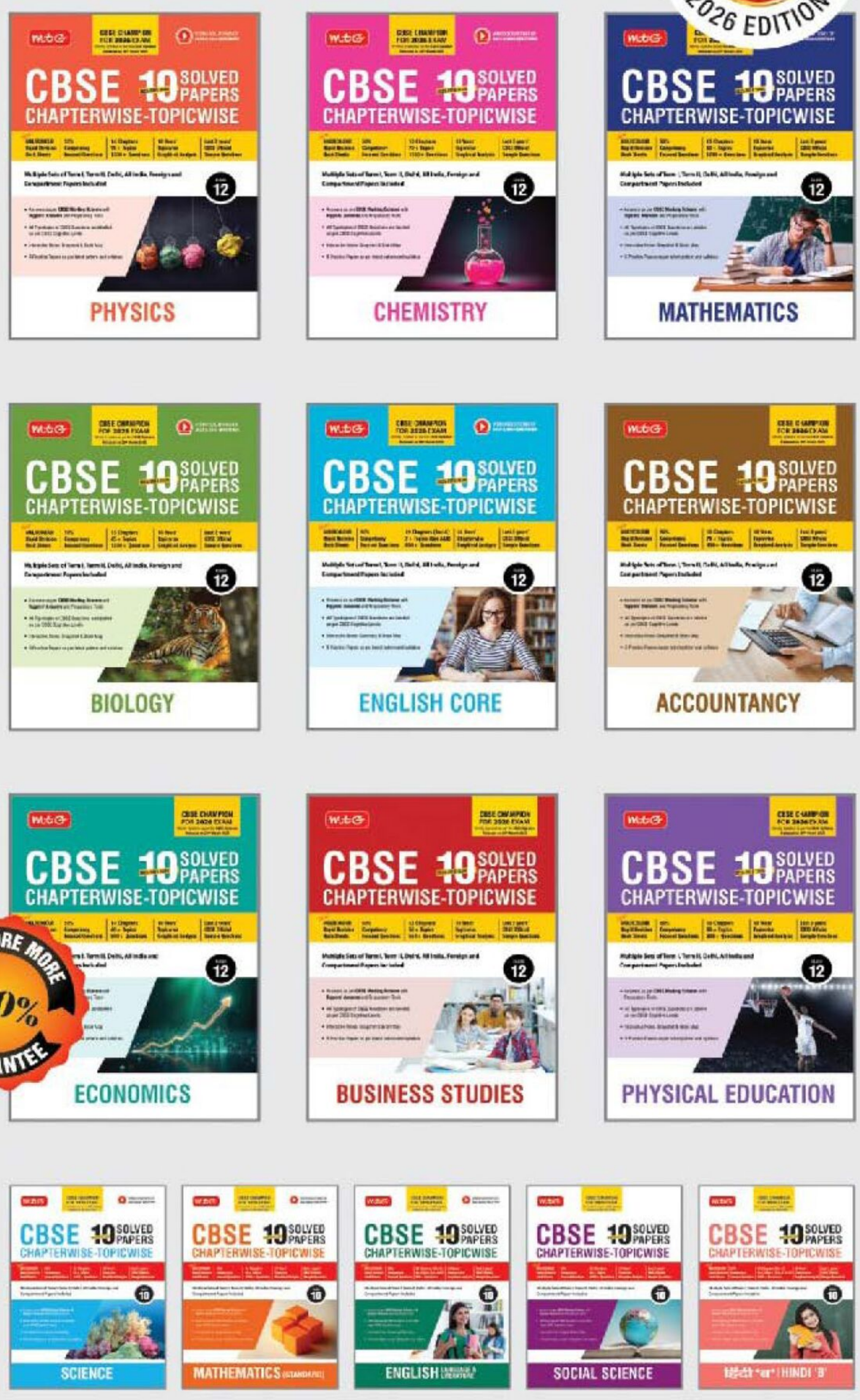


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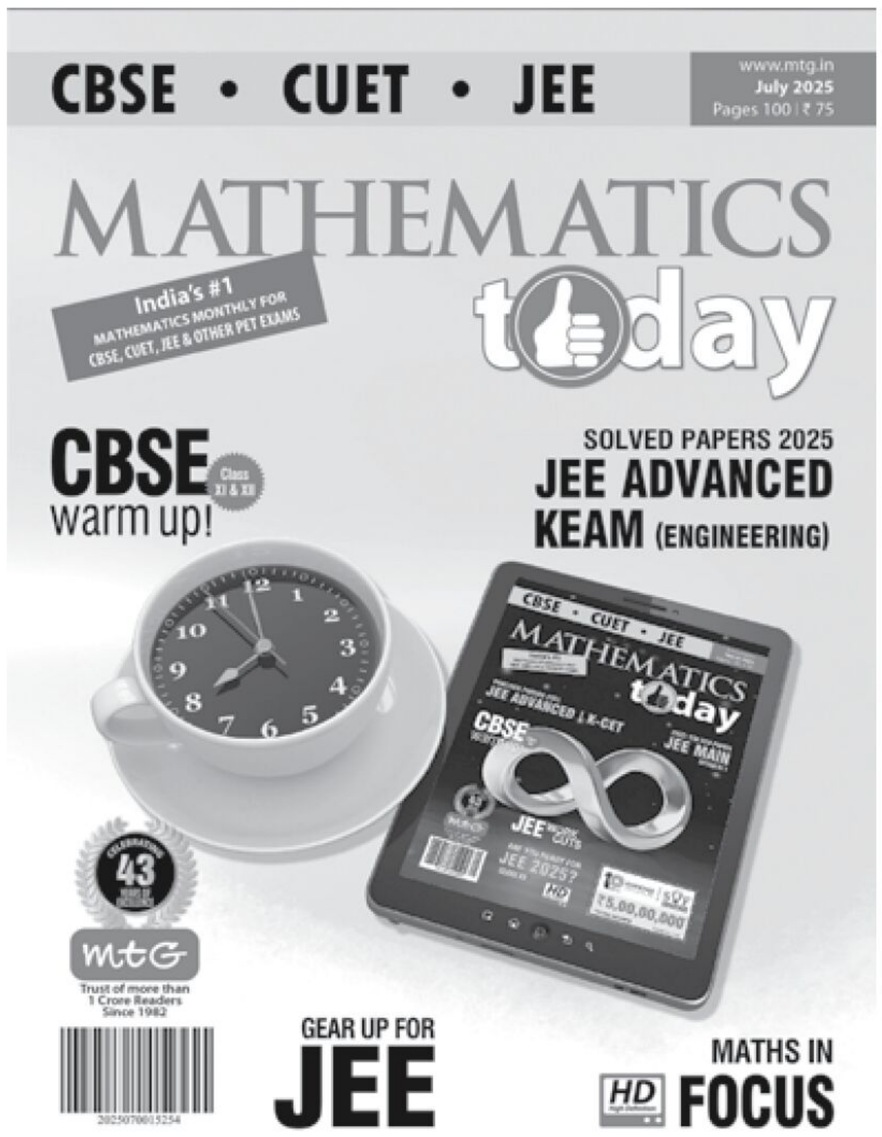



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# Contents


**Competition Edge**

JEE Advanced Solved Paper 2025 9
KEAM Engineering Solved Paper 2025 22
Gear up for JEE 2026 34
Beat the Time Trap 42
Quiz Club 46
Maths in Focus 47
Brain Teaser 55
Quantitative Aptitude 59
GK Corner 63
You Ask We Answer 65
Infinite Learning 66

**Class 11**

CBSE Warm-up! 67
*Trigonometric Functions*
Monthly Test Drive 77
*Trigonometric Functions, Complex Numbers and Quadratic Equations*
Concept Map 50
*Trigonometric Functions*

**Class 12**

CBSE Warm-up! 79
*Matrices & Determinants*
Monthly Test Drive 91
*Inverse Trigonometric Functions*
Concept Map 51
*Inverse Trigonometric Functions*

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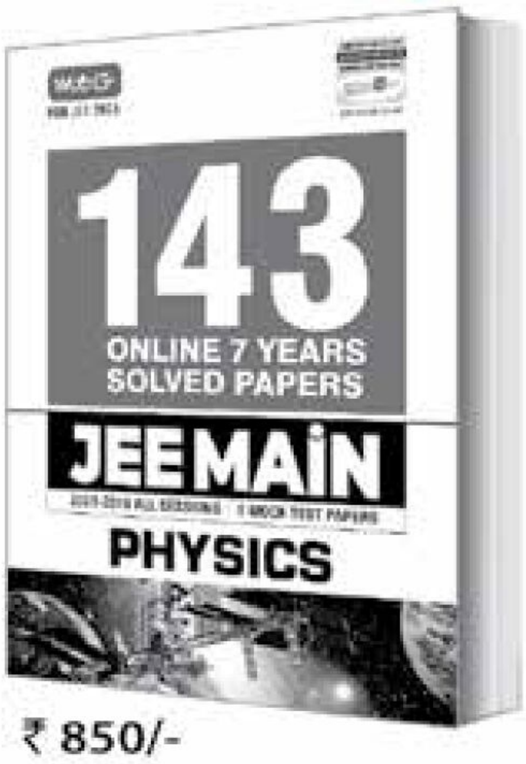


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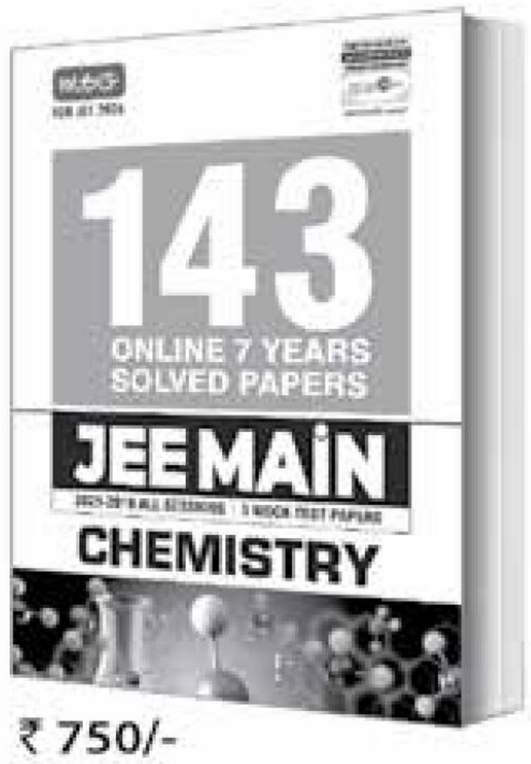


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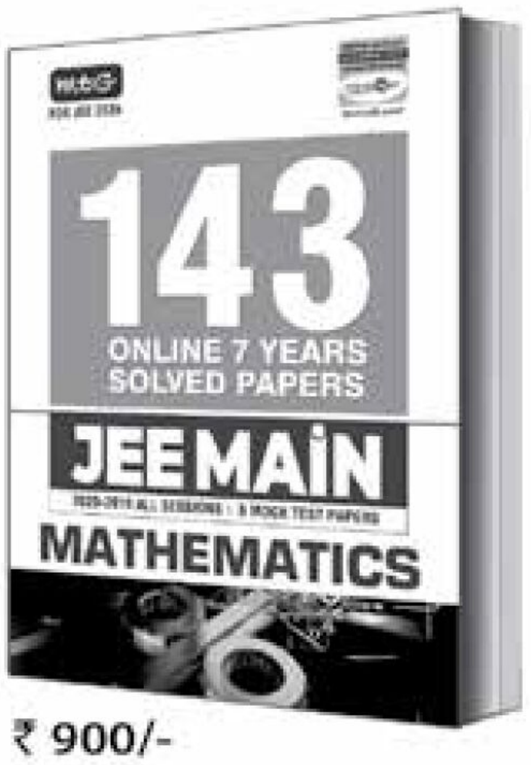


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# JEE 2025 SOLVED PAPER ADVANCED

Exam Held on 18th May

## PAPER-I

### SECTION 1 (MAXIMUM MARKS : 12)

- *This section contains FOUR (04) questions.*
- *Each question has FOUR options (a), (b), (c) and (d). ONLY ONE of these four options is the correct answer.*
- *For each question, choose the option corresponding to the correct answer.*
- *Answer to each question will be evaluated according to the following marking scheme :*

| | | | |
|---|---|---|---|
| *Full Marks* | : | *+3* | *If ONLY the correct option is chosen;* |
| *Zero Marks* | : | *0* | *If none of the options is chosen (i.e. the question is unanswered);* |
| *Negative Marks* | : | *–1* | *In all other cases.* |

**1.** Let $\mathbb{R}$ denote the set of all real numbers. Let $a_i, b_i \in \mathbb{R}$ for $i \in \{1, 2, 3\}$. Define the functions $f: \mathbb{R} \to \mathbb{R}$, $g: \mathbb{R} \to \mathbb{R}$, and $h: \mathbb{R} \to \mathbb{R}$ by

$f(x) = a_1 + 10x + a_2x^2 + a_3x^3 + x^4$,
$g(x) = b_1 + 3x + b_2x^2 + b_3x^3 + x^4$,
$h(x) = f(x + 1) - g(x + 2)$.

If $f(x) \neq g(x)$ for every $x \in \mathbb{R}$, then the coefficient of $x^3$ in $h(x)$ is

(a) 8 (b) 2 (c) –4 (d) –6

**2.** Three students $S_1$, $S_2$, and $S_3$ are given a problem to solve. Consider the following events:

$U$ : At least one of $S_1$, $S_2$, and $S_3$ can solve the problem,

$V$ : $S_1$ can solve the problem, given that neither $S_2$ nor $S_3$ can solve the problem,

$W$ : $S_2$ can solve the problem and $S_3$ cannot solve the problem,

$T$ : $S_3$ can solve the problem.

For any event $E$, let $P(E)$ denote the probability of $E$. If $P(U) = \frac{1}{2}, P(V) = \frac{1}{10}$ and $P(W) = \frac{1}{12}$, then $P(T)$ is equal to

(a) $\frac{13}{36}$ (b) $\frac{1}{3}$ (c) $\frac{19}{60}$ (d) $\frac{1}{4}$

**3.** Let $\mathbb{R}$ denote the set of all real numbers. Define the function $f: \mathbb{R} \to \mathbb{R}$ by

$$f(x) = \begin{cases} 2 - 2x^2 - x^2 \sin\frac{1}{x} & \text{if } x \neq 0, \\ 2 & \text{if } x = 0. \end{cases}$$

Then which one of the following statements is TRUE?

(a) The function $f$ is **NOT** differentiable at $x = 0$

(b) There is a positive real number $\delta$, such that $f$ is a decreasing function on the interval $(0, \delta)$

(c) For any positive real number $\delta$, the function $f$ is **NOT** an increasing function on the interval $(-\delta, 0)$

(d) $x = 0$ is a point of local minima of $f$

**4.** Consider the matrix $P = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$.

Let the transpose of a matrix $X$ be denoted by $X^T$. Then the number of $3 \times 3$ invertible matrices $Q$ with integer entries, such that $Q^{-1} = Q^T$ and $PQ = QP$, is

(a) 32 (b) 8 (c) 16 (d) 24

### SECTION 2 (MAXIMUM MARKS : 12)

- *This section contains THREE (03) questions.*
- *Each question has FOUR options (a), (b), (c) and (d). ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).*
- *For each question, choose the option(s) corresponding to (all) the correct answer(s).*
- *Answer to each question will be evaluated according to the following marking scheme :*

| | | | |
|---|---|---|---|
| *Full Marks* | : | *+4* | *ONLY if (all) the correct option(s) is(are) chosen;* |
| *Partial Marks* | : | *+3* | *If all the four options are correct but ONLY three options are chosen;* |

*Alok Kumar, a B.Tech from IIT Kanpur and INMO 4th ranker of his time, has been training IIT and Olympiad aspirants for close to two decades now. His students have bagged AIR 1 in IIT JEE and also won medals for the country at IMO. He has also taught at Maths Olympiad programme at Cornell University, USA and UT, Dallas. He has been regularly proposing problems in international Mathematics journals.

*Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct;*
*Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;*
*Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);*
*Negative Marks : –2 In all other cases.*

- *For example, in a question, if (a), (b) and (d) are the ONLY three options corresponding to correct answers, then choosing ONLY (a), (b) and (d) will get +4 marks; choosing ONLY (a) and (b) will get +2 marks; choosing ONLY (a) and (d) will get +2 marks; choosing ONLY (b) and (d) will get +2 marks; choosing ONLY (a) will get +1 mark; choosing ONLY (b) will get +1 mark; choosing ONLY (d) will get +1 mark; choosing no option (i.e., the question is unanswered) will get 0 marks; and choosing any other combination of options will get –2 marks.*

**5.** Let $L_1$ be the line of intersection of the planes given by the equations $2x+3y+z=4$ and $x+2y+z=5$. Let $L_2$ be the line passing through the point $P(2, -1, 3)$ and parallel to $L_1$. Let $M$ denote the plane given by the equation $2x+y-2z=6$. Suppose that the line $L_2$ meets the plane $M$ at the point $Q$. Let $R$ be the foot of the perpendicular drawn from $P$ to the plane $M$.
Then which of the following statements is (are) TRUE?

(a) The length of the line segment $PQ$ is $9\sqrt{3}$
(b) The length of the line segment $QR$ is 15
(c) The area of $\Delta PQR$ is $\frac{3}{2}\sqrt{234}$
(d) The acute angle between the line segments $PQ$ and $PR$ is $\cos^{-1}\left(\frac{1}{2\sqrt{3}}\right)$

**6.** Let $\mathbb{N}$ denote the set of all natural numbers, and $\mathbb{Z}$ denote the set of all integers. Consider the functions $f:\mathbb{N}\to\mathbb{Z}$ and $g:\mathbb{Z}\to\mathbb{N}$ defined by

$$f(n)=\begin{cases}(n+1)/2 & \text{if } n \text{ is odd,}\\ (4-n)/2 & \text{if } n \text{ is even,}\end{cases}$$

$$\text{and } g(n)=\begin{cases}3+2n & \text{if } n\geq 0,\\ -2n & \text{if } n<0.\end{cases}$$

Define $(gof)(n)=g(f(n))$ for all $n\in\mathbb{N}$, and $(fog)(n)=f(g(n))$ for all $n\in\mathbb{Z}$.
Then which of the following statements is (are) TRUE?

(a) $gof$ is **NOT** one-one and $gof$ is **NOT** onto
(b) $fog$ is **NOT** one-one and $fog$ is onto
(c) $g$ is one-one and $g$ is onto
(d) $f$ is **NOT** one-one but $f$ is onto

**7.** Let $\mathbb{R}$ denote the set of all real numbers. Let $z_1=1+2i$ and $z_2=3i$ be two complex numbers, where $i=\sqrt{-1}$. Let

$$S=\{(x,y)\in\mathbb{R}\times\mathbb{R}:|x+iy-z_1|=2|x+iy-z_2|\}.$$

Then which of the following statements is (are) TRUE?

(a) $S$ is a circle with centre $\left(-\frac{1}{3},\frac{10}{3}\right)$
(b) $S$ is a circle with centre $\left(\frac{1}{3},\frac{8}{3}\right)$
(c) $S$ is a circle with radius $\frac{\sqrt{2}}{3}$
(d) $S$ is a circle with radius $\frac{2\sqrt{2}}{3}$

## SECTION 3 (MAXIMUM MARKS : 24)

- *This section contains SIX (06) questions.*
- *The answer to each question is a NUMERICAL VALUE.*
- *For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.*
- *If the numerical value has more than two decimal places, truncate/round-off the value to TWO decimal places.*
- *Answer to each question will be evaluated according to the following marking scheme:*
*Full Marks : +4 If ONLY the correct numerical value is entered in the designated place;*
*Zero Marks : 0 In all other cases.*

**8.** Let the set of all relations $R$ on the set $\{a, b, c, d, e, f\}$, such that $R$ is reflexive and symmetric, and $R$ contains exactly 10 elements, be denoted by $\delta$. Then the number of elements in $\delta$ is_____.

**9.** For any two points $M$ and $N$ in the $XY$-plane, let $\overrightarrow{MN}$ denote the vector from $M$ to $N$, and $\vec{0}$ denote the zero vector. Let $P$, $Q$ and $R$ be three distinct points in the $XY$-plane. Let $S$ be a point inside the triangle $\Delta PQR$ such that $\overrightarrow{SP}+5\overrightarrow{SQ}+6\overrightarrow{SR}=\vec{0}$.
Let $E$ and $F$ be the mid-points of the sides $PR$ and $QR$, respectively. Then the value of

$$\frac{\text{length of the line segment } EF}{\text{length of the line segment } ES} \text{ is } _____ .$$

**10.** Let $S$ be the set of all seven-digit numbers that can be formed using the digits 0, 1 and 2. For example, 2210222 is in $S$, but 0210222 is **NOT** in $S$.
Then the number of elements $x$ in $S$ such that at least one of the digits 0 and 1 appears exactly twice in $x$, is equal to _____.

**11.** Let $\alpha$ and $\beta$ be the real numbers such that

$$\lim_{x\to 0}\frac{1}{x^3}\left(\frac{\alpha}{2}\int_0^x \frac{1}{1-t^2}\,dt+\beta x\cos x\right)=2.$$

Then the value of $\alpha+\beta$ is _____.

**12.** Let $\mathbb{R}$ denote the set of all real numbers. Let $f:\mathbb{R}\to\mathbb{R}$ be a function such that $f(x) > 0$ for all $x\in\mathbb{R}$, and $f(x+y)=f(x)f(y)$ for all $x,\ y\in\mathbb{R}$.
Let the real numbers $a_1, a_2, \ldots, a_{50}$ be in an arithmetic progression. If $f(a_{31}) = 64f(a_{25})$, and $\sum_{i=1}^{50} f(a_i)=3(2^{25}+1)$, then the value of $\sum_{i=6}^{30} f(a_i)$ is _____ .

**13.** For all $x > 0$, let $y_1(x)$, $y_2(x)$, and $y_3(x)$ be the functions satisfying

$$\frac{dy_1}{dx}-(\sin x)^2 y_1=0,\ \ y_1(1)=5,$$
$$\frac{dy_2}{dx}-(\cos x)^2 y_2=0,\ \ y_2(1)=\frac{1}{3},$$
$$\frac{dy_3}{dx}-\left(\frac{2-x^3}{x^3}\right)y_3=0,\ \ y_3(1)=\frac{3}{5e},$$

respectively. Then $\lim\limits_{x\to 0^+}\dfrac{y_1(x)y_2(x)y_3(x)+2x}{e^{3x}\cdot\sin x}$ is equal to _____.

## SECTION 4 (MAXIMUM MARKS : 12)

- ***This section contains THREE (03) Matching List Sets.***
- ***Each set has ONE Multiple Choice Question.***
- ***Each set has TWO lists: List-I and List-II.***
- ***List-I has Four entries (P), (Q), (R) and (S) and List-II has Five entries (1), (2), (3), (4) and (5).***
- ***FOUR options are given in each Multiple Choice Question based on List-I and List-II and ONLY ONE of these four options satisfies the condition asked in the Multiple Choice Question.***
- ***Answer to each question will be evaluated according to the following marking scheme:***

| | | |
|---|---|---|
| ***Full Marks*** | ***: +4*** | ***ONLY if the option corresponding to the correct combination is chosen;*** |
| ***Zero Marks*** | ***: 0*** | ***If none of the options is chosen (i.e. the question is unanswered);*** |
| ***Negative Marks*** | ***: –1*** | ***In all other cases.*** |

**14.** Consider the following frequency distribution:

| Value | 4 | 5 | 8 | 9 | 6 | 12 | 11 |
|---|---|---|---|---|---|---|---|
| Frequency | 5 | $f_1$ | $f_2$ | 2 | 1 | 1 | 3 |

Suppose that the sum of the frequencies is 19 and the median of this frequency distribution is 6.
For the given frequency distribution, let $\alpha$ denote the mean deviation about the mean, $\beta$ denote the mean deviation about the median, and $\sigma^2$ denote the variance.
Match each entry in List-I to the correct entry in List-II and choose the correct option.

| **List-I** | | **List-II** | |
|---|---|---|---|
| (P) | $7f_1+9f_2$ is equal to | (1) | 146 |
| (Q) | $19\alpha$ is equal to | (2) | 47 |
| (R) | $19\beta$ is equal to | (3) | 48 |
| (S) | $19\sigma^2$ is equal to | (4) | 145 |
| | | (5) | 55 |

(a) (P) → (5); (Q) → (3); (R) → (2); (S) → (4)
(b) (P) → (5); (Q) → (2); (R) → (3); (S) → (1)
(c) (P) → (5); (Q) → (3); (R) → (2); (S) → (1)
(d) (P) → (3); (Q) → (2); (R) → (5); (S) → (4)

**15.** Let $\mathbb{R}$ denote the set of all real numbers. For a real number $x$, let $[x]$ denote the greatest integer less than or equal to $x$. Let $n$ denote a natural number. Match each entry in List-I to the correct entry in List-II and choose the correct option.

| **List-I** | | **List-II** | |
|---|---|---|---|
| (P) | The minimum value of $n$ for which the function $f(x)=\left[\dfrac{10x^3-45x^2+60x+35}{n}\right]$ is continuous on the interval [1, 2], is | (1) | 8 |
| (Q) | The minimum value of $n$ for which $g(x)=(2n^2-13n-15)(x^3+3x)$, $x\in\mathbb{R}$, is an increasing function on $\mathbb{R}$, is | (2) | 9 |
| (R) | The smallest natural number $n$ which is greater than 5, such that $x = 3$ is a point of local minima of $h(x)=(x^2-9)^n(x^2+2x+3)$, is | (3) | 5 |

| | | | |
|---|---|---|---|
| (S) | Number of $x_0 \in \mathbb{R}$ such that $l(x)=\sum_{k=0}^{4}\left(\sin\lvert x-k\rvert+\cos\left\lvert x-k+\frac{1}{2}\right\rvert\right)$, $x\in\mathbb{R}$, is NOT differentiable at $x_0$, is | (4) | 6 |
| | | (5) | 10 |

(a) (P) → (1); (Q) → (3); (R) → (2); (S) → (5)
(b) (P) → (2); (Q) → (1); (R) → (4); (S) → (3)
(c) (P) → (5); (Q) → (1); (R) → (4); (S) → (3)
(d) (P) → (2); (Q) → (3); (R) → (1); (S) → (5)

**16.** Let $\vec{w}=\hat{i}+\hat{j}-2\hat{k}$, and $\vec{u}$ and $\vec{v}$ be two vectors, such that $\vec{u}\times\vec{v}=\vec{w}$ and $\vec{v}\times\vec{w}=\vec{u}$. Let $\alpha$, $\beta$, $\gamma$, and $t$ be real numbers such that $\vec{u}=\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$, $-t\alpha+\beta+\gamma=0$, $\alpha-t\beta+\gamma=0$, and $\alpha+\beta-t\gamma=0$. Match each entry in List-I to the correct entry in List-II and choose the correct option.

| List-I | | List-II | |
|---|---|---|---|
| (P) | $\lvert\vec{v}\rvert^2$ is equal to | (1) | 0 |
| (Q) | If $\alpha=\sqrt{3}$, then $\gamma^2$ is equal to | (2) | 1 |
| (R) | If $\alpha=\sqrt{3}$, then $(\beta+\gamma)^2$ is equal to | (3) | 2 |
| (S) | If $\alpha=\sqrt{2}$, then $t+3$ is equal to | (4) | 3 |
| | | (5) | 5 |

(a) (P) → (2); (Q) → (1); (R) → (4); (S) → (5)
(b) (P) → (2); (Q) → (4); (R) → (3); (S) → (5)
(c) (P) → (2); (Q) → (1); (R) → (4); (S) → (3)
(d) (P) → (5); (Q) → (4); (R) → (1); (S) → (3)

## PAPER-II

### SECTION 1 (MAXIMUM MARKS : 12)

- ***This section contains FOUR (04) questions.***
- ***Each question has FOUR options (a), (b), (c) and (d). ONLY ONE of these four options is the correct answer.***
- ***For each question, choose the option corresponding to the correct answer.***
- ***Answer to each question will be evaluated according to the following marking scheme:***

| | | |
|---|---|---|
| ***Full Marks*** | : +3 | ***If ONLY the correct option is chosen;*** |
| ***Zero Marks*** | : 0 | ***If none of the options is chosen (i.e., the question is unanswered);*** |
| ***Negative Marks*** | : −1 | ***In all other cases.*** |

**1.** Let $x_0$ be the real number such that $e^{x_0}+x_0=0$. For a given real number $\alpha$, define $g(x)=\frac{3xe^x+3x-\alpha e^x-\alpha x}{3(e^x+1)}$ for all real numbers $x$. Then which one of the following statements is TRUE?

(a) For $\alpha=2$, $\lim_{x\to x_0}\left\lvert\frac{g(x)+e^{x_0}}{x-x_0}\right\rvert=0$

(b) For $\alpha=2$, $\lim_{x\to x_0}\left\lvert\frac{g(x)+e^{x_0}}{x-x_0}\right\rvert=1$

(c) For $\alpha=3$, $\lim_{x\to x_0}\left\lvert\frac{g(x)+e^{x_0}}{x-x_0}\right\rvert=0$

(d) For $\alpha=3$, $\lim_{x\to x_0}\left\lvert\frac{g(x)+e^{x_0}}{x-x_0}\right\rvert=\frac{2}{3}$

**2.** Let $\mathbb{R}$ denote the set of all real numbers. Then the area of the region $\{(x,y)\in\mathbb{R}\times\mathbb{R}: x>0, y>\frac{1}{x}, 5x-4y-1>0, 4x+4y-17>0\}$ is

(a) $\frac{17}{16}-\log_e 4$ (b) $\frac{33}{8}-\log_e 4$

(c) $\frac{57}{8}-\log_e 4$ (d) $\frac{17}{2}-\log_e 4$

**3.** The total number of real solutions of the equation $\theta=\tan^{-1}(2\tan\theta)-\frac{1}{2}\sin^{-1}\left(\frac{6\tan\theta}{9+\tan^2\theta}\right)$ is

(Here, the inverse trigonometric functions $\sin^{-1}x$ and $\tan^{-1}x$ assume values in $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ and $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$, respectively.)

(a) 1 (b) 2 (c) 3 (d) 5

**4.** Let $S$ denote the locus of the point of intersection of the pair of lines $4x-3y=12\alpha$, $4\alpha x+3\alpha y=12$, where $\alpha$ varies over the set of non-zero real numbers. Let $T$ be the tangent to $S$ passing through the points $(p, 0)$ and $(0, q)$, $q>0$, and parallel to the line $4x-\frac{3}{\sqrt{2}}y=0$. Then the value of $pq$ is

(a) $-6\sqrt{2}$ (b) $-3\sqrt{2}$ (c) $-9\sqrt{2}$ (d) $-12\sqrt{2}$

## SECTION 2 (MAXIMUM MARKS : 16)

- ***This section contains FOUR (04) questions.***
- ***Each question has FOUR options (a), (b), (c) and (d). ONE OR MORE THAN ONE of these four option(s) is(are) correct answer(s).***
- ***For each question, choose the option(s) corresponding to (all) the correct answer(s).***
- ***Answer to each question will be evaluated according to the following marking scheme:***

| | | | |
|---|---|---|---|
| ***Full Marks*** | ***:*** | ***+4*** | ***ONLY if (all) the correct option(s) is(are) chosen;*** |
| ***Partial Marks*** | ***:*** | ***+3*** | ***If all the four options are correct but ONLY three options are chosen;*** |
| ***Partial Marks*** | ***:*** | ***+2*** | ***If three or more options are correct but ONLY two options are chosen, both of which are correct;*** |
| ***Partial Marks*** | ***:*** | ***+1*** | ***If two or more options are correct but ONLY one option is chosen and it is a correct option;*** |
| ***Zero Marks*** | ***:*** | ***0*** | ***If none of the options is chosen (i.e., the question is unanswered);*** |
| ***Negative Marks*** | ***:*** | ***–2*** | ***In all other cases.*** |

- ***For example, in a question, if (a), (b) and (d) are the ONLY three options corresponding to correct answers, then choosing ONLY (a), (b) and (d) will get +4 marks; choosing ONLY (a) and (b) will get +2 marks; choosing ONLY (a) and (d) will get +2 marks; choosing ONLY (b) and (d) will get +2 marks; choosing ONLY (a) will get +1 mark; choosing ONLY (b) will get +1 mark; choosing ONLY (d) will get +1 mark; choosing no option (i.e., the question is unanswered) will get 0 marks; and choosing any other combination of options will get –2 marks.***

**5.** Let $I=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$ and $P=\begin{pmatrix}2 & 0\\ 0 & 3\end{pmatrix}$. Let $Q=\begin{pmatrix}x & y\\ z & 4\end{pmatrix}$ for some non-zero real numbers $x$, $y$, and $z$, for which there is a $2 \times 2$ matrix $R$ with all entries being non-zero real numbers, such that $QR = RP$. Then which of the following statements is (are) TRUE?

(a) The determinant of $Q - 2I$ is zero

(b) The determinant of $Q - 6I$ is 12

(c) The determinant of $Q - 3I$ is 15

(d) $yz = 2$

**6.** Let $S$ denote the locus of the mid-points of those chords of the parabola $y^2 = x$, such that the area of the region enclosed between the parabola and the chord is $\frac{4}{3}$. Let $R$ denote the region lying in the first quadrant, enclosed by the parabola $y^2 = x$, the curve $S$, and the lines $x = 1$ and $x = 4$. Then which of the following statements is (are) TRUE?

(a) $(4, \sqrt{3}) \in S$ (b) $(5, \sqrt{2}) \in S$

(c) Area of $R$ is $\frac{14}{3} - 2\sqrt{3}$

(d) Area of $R$ is $\frac{14}{3} - \sqrt{3}$

**7.** Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be two distinct points on the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ such that $y_1 > 0$, and $y_2 > 0$. Let $C$ denote the circle $x^2 + y^2 = 9$, and $M$ be the point (3, 0). Suppose the line $x = x_1$ intersects $C$ at $R$, and the line $x = x_2$ intersects $C$ at $S$, such that the $y$-coordinates of $R$ and $S$ are positive. Let $\angle ROM = \frac{\pi}{6}$ and $\angle SOM = \frac{\pi}{3}$, where $O$ denotes the origin (0, 0). Let $|XY|$ denote the length of the line segment $XY$. Then which of the following statements is (are) TRUE?

(a) The equation of the line joining $P$ and $Q$ is $2x + 3y = 3(1+\sqrt{3})$

(b) The equation of the line joining $P$ and $Q$ is $2x + y = 3(1+\sqrt{3})$

(c) If $N_2 = (x_2, 0)$, then $3|N_2Q| = 2|N_2S|$

(d) If $N_1 = (x_1, 0)$, then $9|N_1P| = 4|N_1R|$

**8.** Let $\mathbb{R}$ denote the set of all real numbers. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x)=\begin{cases}\dfrac{6x+\sin x}{2x+\sin x} & \text{if } x \neq 0,\\ \dfrac{7}{3} & \text{if } x = 0.\end{cases}$$

Then which of the following statements is (are) TRUE?

(a) The point $x = 0$ is a point of local maxima of $f$

(b) The point $x = 0$ is a point of local minima of $f$

(c) Number of points of local maxima of $f$ in the interval $[\pi, 6\pi]$ is 3

(d) Number of points of local minima of $f$ in the interval $[2\pi, 4\pi]$ is 1

## SECTION 3 (MAXIMUM MARKS : 32)

- ***This section contains EIGHT (08) questions.***
- ***The answer to each question is a NUMERICAL VALUE.***
- ***For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.***
- ***If the numerical value has more than two decimal places, truncate/round-off the value to TWO decimal places.***
- ***Answer to each question will be evaluated according to the following marking scheme:***

***Full Marks*** : ***+4 If ONLY the correct numerical value is entered in the designated place;***
***Zero Marks*** : ***0 In all other cases.***

**9.** Let $y(x)$ be the solution of the differential equation $x^2\frac{dy}{dx}+xy=x^2+y^2, x>\frac{1}{e}$, satisfying $y(1)=0$.

Then the value of $2\frac{(y(e))^2}{y(e^2)}$ is _____.

**10.** Let $a_0, a_1,\ldots, a_{23}$ be real numbers such that

$$\left(1+\frac{2}{5}x\right)^{23}=\sum_{i=0}^{23}a_i x^i \text{ for every real number } x.$$

Let $a_r$ be the largest among the numbers $a_j$ for $0\le j\le 23$. Then the value of $r$ is _____.

**11.** A factory has a total of three manufacturing units, $M_1$, $M_2$, and $M_3$, which produce bulbs independent of each other. The units $M_1$, $M_2$, and $M_3$ produce bulbs in the proportions of 2 : 2 : 1, respectively. It is known that 20% of the bulbs produced in the factory are defective. It is also known that, of all the bulbs produced by $M_1$, 15% are defective. Suppose that, if a randomly chosen bulb produced in the factory is found to be defective, the probability that it was produced by $M_2$ is 2/5. If a bulb is chosen randomly from the bulbs produced by $M_3$, then the probability that it is defective is _____.

**12.** Consider the vectors $\vec{x}=\hat{i}+2\hat{j}+3\hat{k}$, $\vec{y}=2\hat{i}+3\hat{j}+\hat{k}$, and $\vec{z}=3\hat{i}+\hat{j}+2\hat{k}$. For two distinct positive real numbers $\alpha$ and $\beta$, define $\vec{X}=\alpha\vec{x}+\beta\vec{y}-\vec{z}$, $\vec{Y}=\alpha\vec{y}+\beta\vec{z}-\vec{x}$, and $\vec{Z}=\alpha\vec{z}+\beta\vec{x}-\vec{y}$.

If the vectors $\vec{X},\vec{Y}$ and $\vec{Z}$ lie in a plane, then the value of $\alpha+\beta-3$ is _____.

**13.** For a non-zero complex number $z$, let $\arg(z)$ denote the principal argument of $z$, with $-\pi<\arg(z)\le\pi$. Let $\omega$ be the cube root of unity for which $0<\arg(\omega)<\pi$. Let $\alpha=\arg\left(\sum_{n=1}^{2025}(-\omega)^n\right)$. Then the value of $\frac{3\alpha}{\pi}$ is _____.

**14.** Let $\mathbb{R}$ denote the set of all real numbers. Let $f:\mathbb{R}\to\mathbb{R}$ and $g:\mathbb{R}\to(0,4)$ be functions defined by $f(x)=\log_e(x^2+2x+4)$, and $g(x)=\frac{4}{1+e^{-2x}}$. Define the composite function $fog^{-1}$ by $(fog^{-1})(x)=f(g^{-1}(x))$, where $g^{-1}$ is the inverse of the function $g$. Then the value of the derivative of the composite function $fog^{-1}$ at $x=2$ is _____.

**15.** Let $\alpha=\frac{1}{\sin 60^\circ\sin 61^\circ}+\frac{1}{\sin 62^\circ\sin 63^\circ}+\ldots.+\frac{1}{\sin 118^\circ\sin 119^\circ}$.

Then the value of $\left(\frac{\text{cosec}1^\circ}{\alpha}\right)^2$ is _____.

**16.** If $\alpha=\int_{1/2}^{2}\frac{\tan^{-1}x}{2x^2-3x+2}dx$, then the value of

$\sqrt{7}\tan\left(\frac{2\alpha\sqrt{7}}{\pi}\right)$ is _____.

(Here, the inverse trigonometric function $\tan^{-1}x$ assumes values in $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$.

## SOLUTIONS

### PAPER-I

**1. (c) :** We have, $h(x)=f(x+1)-g(x+2)$

We are concerned with coefficient of $x^3$ in $h(x)$. Now, coefficient of $x^3$ in $f(x+1)$ – coefficient of $x^3$ in $g(x+2)$ = coefficient of $x^3$ in $((x+1)^4+a_3(x+1)^3)$ – coefficient of $x^3$ in $((x+2)^4+b_3(x+2)^3)=(a_3+4)-(b_3+8)$ $=a_3-b_3-4$

Also, $f(x)-g(x)=(a_3-b_3)x^3+(a_2-b_2)x^2+7x+(a_1-b_1)$

According to question, $f(x)-g(x)\neq 0$ *i.e.*, it has no roots.

$\Rightarrow$ It is not a cubic but may be a quadratic.

$\Rightarrow (a_3-b_3)=0$

$\therefore\ a_3-b_3-4=0-4=-4$

**2. (a) :** Let $P(S_1)=a$, $P(S_2)=b$ and $P(S_3)=c$

They independently solving the problem.

$$U=S_1\cup S_2\cup S_3\ \Rightarrow\ P(U)=P(S_1\cup S_2\cup S_3)=\frac{1}{2}$$

$$P(\text{V})=P\left(\frac{S_1}{S_2^c\cap S_3^c}\right)=\frac{P(S_1\cap S_2^c\cap S_3^c)}{P(S_2^c\cap S_3^c)}=P(S_1)=\frac{1}{10}=a$$

and $P(W)=P(S_2\cap S_3^c)=\frac{1}{12}=b(1-c)$ ...(i)

Now, $P(U^c)=1-P(U)=1-\frac{1}{2}\Rightarrow P(S_1^c\cap S_2^c\cap S_3^c)=\frac{1}{2}$

$\Rightarrow\ (1-a)(1-b)(1-c)=\frac{1}{2}\ \Rightarrow\ \frac{9}{10}(1-b)(1-c)=\frac{1}{2}$

$\Rightarrow \quad (1-b)(1-c)=\dfrac{5}{9}$

Divide (i) by (ii), we get

$$\frac{b}{1-b}=\frac{3}{20} \quad \Rightarrow \quad 20b = 3 - 3b \quad \Rightarrow \quad b=\frac{3}{23}$$

Put $b = 3/23$ in (i), we get

$$\Rightarrow \quad c=\frac{13}{36} \quad \therefore \quad P(T)=P(S_3)=c=\frac{13}{36}$$

**3. (b) :** We have, $f(x)=2-x^2\left(2+\sin\left(\dfrac{1}{x}\right)\right)$ if $x \neq 0$

Now, $2+\sin\left(\dfrac{1}{x}\right)>0 \quad \Rightarrow \quad x^2\left(2+\sin\left(\dfrac{1}{x}\right)\right)>0 \quad x \neq 0$

Checking continuity : $f(0)=2, \lim\limits_{x\to 0} f(x)=2$

$$f'(0)=\lim_{h\to 0}\frac{2-h^2\left(2+\sin\frac{1}{h}\right)-2}{h}$$

$$=\lim_{h\to 0}\frac{-h\left(2+\sin\frac{1}{h}\right)}{1}=0$$

$\Rightarrow \quad f'(0) = 0$

$\therefore \quad f(x)$ is differentiable at $x = 0$.

At all values except 0, $f(x) < 2$ as $x^2\left(2+\sin\left(\dfrac{1}{x}\right)\right)>0$

$\Rightarrow \quad x = 0$ is a point of maxima.

Then, there exist $\delta > 0$ for which $f'(-\delta) > 0$ for $-\delta < 0$ and $f'(\delta) < 0$ for $\delta < 0$. So, $f$ is decreasing function on the interval $(0, \delta)$ and $f$ is an increasing function on the interval $(-\delta, 0)$.

**4. (c) :** Let $Q = [a_{ij}]$, where, $1 \le i, j \le 3 \ \forall \ i, j \in$ N

$$PQ=\begin{bmatrix}2a_{11} & 2a_{12} & 2a_{13}\\ 2a_{21} & 2a_{22} & 2a_{23}\\ 3a_{31} & 3a_{32} & 3a_{33}\end{bmatrix} \quad QP=\begin{bmatrix}2a_{11} & 2a_{12} & 3a_{13}\\ 2a_{21} & 2a_{22} & 3a_{23}\\ 2a_{31} & 2a_{32} & 3a_{33}\end{bmatrix}$$

$\Rightarrow \quad a_{13} = a_{23} = a_{31} = a_{32} = 0$

$$\Rightarrow \quad Q=\begin{bmatrix}a_{11} & a_{12} & 0\\ a_{21} & a_{22} & 0\\ 0 & 0 & a_{33}\end{bmatrix}=\begin{bmatrix}a & b & 0\\ c & d & 0\\ 0 & 0 & e\end{bmatrix}$$

Now, $QQ^T = I$

$$\Rightarrow \quad \begin{bmatrix}a & b & 0\\ c & d & 0\\ 0 & 0 & e\end{bmatrix}\begin{bmatrix}a & c & 0\\ b & d & 0\\ 0 & 0 & e\end{bmatrix}=\begin{bmatrix}a^2+b^2 & ac+bd & 0\\ ac+bd & c^2+d^2 & 0\\ 0 & 0 & e^2\end{bmatrix}$$

$\Rightarrow \quad a^2 + b^2 = 1, c^2 + d^2 = 1, \ e^2 = 1, \ ac + bd = 0$

Case 1 : $(a, b) = (0, \pm 1) \Rightarrow (c, d) = (\pm 1, 0), e = \pm 1$

$\therefore$ Number of way $= 2 \times 2 \times 2 = 8$

Case 2 : $(a, b) = (\pm 1, 0) \Rightarrow (c, d) = (0, \pm 1), e = \pm 1$

$\therefore$ Number of way $= 2 \times 2 \times 2 = 8$

$\therefore$ Total number of way = 16

**5. (a,c) :** D.R's of $L_1 = \langle 1, -1, 1\rangle$

$$\therefore \quad L_2 : \frac{x-2}{1}=\frac{y+1}{-1}=\frac{z-3}{1}=K$$

$\Rightarrow$ Point $Q(K + 2, -K, -1, K + 3)$ is on $L_1$

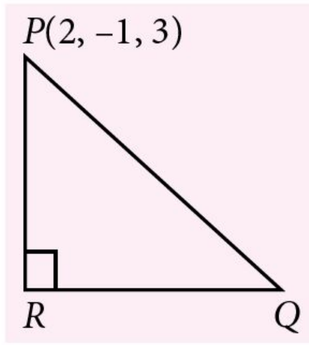
$Q$ is on $L_2$ and plane $M$.

$\Rightarrow \quad 2K + 4 - K - 1 - 2K - 6 = 6$

$\Rightarrow \quad K = -9$

$\therefore \quad Q \equiv (-7, 8, -6)$

$$\overrightarrow{PQ}=\sqrt{9^2+9^2+9^2}=\sqrt{243}=9\sqrt{3}$$




and $\overrightarrow{PR}=\left|\dfrac{2(2)+(-1)-2(3)-6}{\sqrt{4+1+4}}\right|=3$

By Pythagoras theorem, $\overrightarrow{QR}=\sqrt{234}$

Area $(PQR)=\dfrac{1}{2}\times\overline{PR}\times\overline{QR}=\dfrac{3}{2}\sqrt{234}$ sq. units.

Angle between $PQ$ and $PR=\cos^{-1}\left(\dfrac{\overline{PR}}{\overline{PQ}}\right)=\cos^{-1}\left(\dfrac{1}{3\sqrt{3}}\right)$

**6. (a,d) :** $f(n)=\begin{cases}K & \text{if } n=2K-1\\ 2-K & \text{if } n=2K\end{cases} \forall\ K\in N$

Range of $f(n) \in Z$ as $K$ and $2 - K$ cover $Z$ with just $N$ as inputs $\Rightarrow f$ is onto.

Observe that $f(1) = f(2) = 1 \Rightarrow f$ is not one-one.

$$g(n)=\begin{cases}3+2n & \text{if} \quad n\ge 0\\ -2n & \text{if} \quad n<0\end{cases}$$

$\Rightarrow \quad gof(1) = gof(2) \Rightarrow gof$ is not one-one

Observe that $3 + 2n \ge 3 \ \forall \ n \ge 0$ and $-2n \le -2 \ \forall \ n \le -1$

$\Rightarrow$ 1 is not in range of $g(n) \Rightarrow g(n)$ is not onto.

$\Rightarrow$ $gof$ is not onto.

$$\Rightarrow \quad fog(n)=\begin{cases}\dfrac{g(n)+1}{2}, & g(n) \text{ is odd}\\ \dfrac{4-g(n)}{2}, & g(n) \text{ is even}\end{cases}$$

$$=\begin{cases}\dfrac{3+2n+1}{2}=n+2 \ \forall\ n\ge 0\\ \dfrac{4-(-2n)}{2}=n+2 \ \forall\ n<0\end{cases}$$

Thus, $fog(n) = n + 2 \ \forall \ n \in Z$

$\therefore$ $fog(n)$ is one-one and onto.

**7. (a,d) :** $|x + iy - 1 - 2i| = 2\,|x + iy - 3i|$

$$\Rightarrow \sqrt{(x-1)^2+(y-2)^2} = \sqrt{4x^2+4(y-3)^2}$$

$$\Rightarrow 3x^2 + 3y^2 + 2x - 20y + 31 = 0$$

$$\Rightarrow x^2+y^2+\frac{2}{3}x-\frac{20}{3}y+\frac{31}{3}=0$$

$$\therefore \quad \text{Center} = \left(-\frac{1}{3}, \frac{10}{3}\right)$$

$$\therefore \quad \text{Radius} = \sqrt{\frac{1}{9}+\frac{100}{9}-\frac{31}{3}} = \frac{2\sqrt{2}}{3}$$

**8. (105) :** 6 out of the 10 elements must be $(a, a)$, $(b, b)$, ....., $(f, f)$ for reflexive

For the remaining 4 spots, we can choose two pairs of form $(a, b)$ and $(b, a) \in R$

Total Number of pairs to pick = ${}^6C_2 = 15$

Out of these number of ways to pick two pairs = ${}^{15}C_2$ $= 15 \times 7 = 105$

**9. (1.2) :** Let position vectors of $P, Q, R, S$ be $\vec{p}, \vec{q}, \vec{r}, \vec{s}$

$$E = \frac{\vec{p}+\vec{r}}{2}, \ F = \frac{\vec{q}+\vec{r}}{2}$$

$$SP + 5SQ + 6SR = 0 \ \Rightarrow \ \vec{p} - \vec{s} + 5(\vec{q} - \vec{s}) + 6(\vec{r} - \vec{s}) = 0$$

$$\Rightarrow \ \vec{s} = \frac{\vec{p} + 5\vec{q} + 6\vec{r}}{12}$$

$$\therefore \quad |\overrightarrow{ES}| = \left|\frac{5\vec{q} - 5\vec{p}}{12}\right| \text{ and } |\overrightarrow{EF}| = \left|\frac{\vec{q} - \vec{p}}{2}\right|$$

$$\therefore \quad \frac{|\overrightarrow{EF}|}{|\overrightarrow{ES}|} = \frac{1/2}{5/12} = \frac{6}{5} = 1.2$$

**10. (762) :** Let $A = 0$ appears twice exactly and $B = 1$ appears exactly twice.

$A$ : 0 may go in any 2 spots except first one and remaining 5 spots have two choices each.

$\Rightarrow \ n(A) = {}^6C_2 \times 2^5 = 480$

For $B$ :

Case I : 1 at first place, 1 more one is remaining 6 spots and left 5 have 2 choices

$= {}^6C_1 \times 2^5 = 192$

Case II : 2 at first place, 2 out of 6 spots for 1, remaining 4 for 0 or 2

$\therefore$ Number of choices $= {}^6C_2 \times 2^4 = 240 \Rightarrow n(B) = 432$

Now, $A \cap B$ : 2 spots for 0 except first, 2 spots for 1 in remaining 5, rest are all 2 only $= {}^6C_2 \times {}^5C_2 = 150$

$\therefore \quad n(A \cup B) = 480 + 432 - 150 = 762$

**11. (2.4) :**
$$\lim_{x \to 0} \frac{\frac{\alpha}{2}\int_0^x \frac{dt}{1-t^2} + \beta x \cos x}{3x^2}$$

$$= \lim_{x \to 0} \frac{\frac{\alpha}{2}\frac{1}{1-x^2} + \beta\,(\cos x - x \sin x)}{3x^2}$$

$$= \lim_{x \to 0} \frac{\frac{\alpha}{2}(1+x^2) + \beta\left(1 - \frac{x^2}{2} - x(x)\right)}{3x^2}$$

$$= \lim_{x \to 0} \left(\frac{\frac{\alpha}{2} + \beta}{3x^2} + \frac{\alpha - 3\beta}{6}\right) = 2$$

$$\Rightarrow \ \frac{\alpha}{2} + \beta = 0, \frac{\alpha - 3\beta}{6} = 2$$

$$\Rightarrow \ \alpha = -2\beta, \alpha - 3\beta = 12 \ \Rightarrow \ -5\beta = 12 \ \Rightarrow \ \beta = -\frac{12}{5}$$

$$\alpha + \beta = -\beta = \frac{12}{5}$$

**12. (96) :** $f(x + y) = f(x)\, f(y)$

$$\Rightarrow \ f(x) = K^x \qquad (\because \ f(x) > 0 \ \forall \ x \in \mathbb{R})$$

Now, $f(a_{31}) = 64 f(a_{25})$

$$\Rightarrow \ K^{a_{31}} = 64K^{a_{25}} \ \Rightarrow \ K^{a_{31} - a_{25}} = 2^6 \ \Rightarrow \ K^{6d} = 2^6$$

$$\Rightarrow \ K^d = 2$$

Also, $\sum_1^{50} f(a_i) = K^{a_1} + K^{a_1 + d} ..... + K^{a_1 + 49d}$

$$= \frac{K^{a_1}(K^{50d} - 1)}{K^d - 1} = K^{a_1}(2^{50} - 1) = 3(2^{25} + 1) \qquad \text{[Given]}$$

$$\Rightarrow \ K^{a_1} = \frac{3(2^{25}+1)}{(2^{25}+1)(2^{25}-1)} = \frac{3}{2^{25}-1}$$

Now, $\sum_6^{30} f(a_i) = K^{a_1 + 5d} + K^{a_1 + 6d} + .... + K^{a_1 + 29d}$

$$= \frac{K^{a_1 + 5d}(K^{25d} - 1)}{K^d - 1} = K^{a_1} \times 2^5 \times (2^{25} - 1) = 3 \times 2^5 = 96$$

**13. (2) :** $\frac{dy_1}{y_1} + \frac{dy_2}{y_2} + \frac{dy_3}{y_3}$

$$= \left(\sin^2 x + \cos^2 x + \frac{2 - x^3}{x^3}\right)dx = \frac{2}{x^3}\,dx$$

$$\Rightarrow \ \ln(y_1) + \ln(y_2) + \ln(y_3) = \frac{-1}{x^2} + c$$

$$\Rightarrow \ \ln(y_1 y_2 y_3) = \frac{-1}{x^2} + c$$

At $x=1: \ln\left(\frac{1}{e}\right) = -\frac{1}{1} + c \Rightarrow c = 0 \Rightarrow y_1 y_2 y_3 = e^{-1/x^2}$

Now, $\lim\limits_{x\to 0} \dfrac{e^{-\frac{1}{x^2}} + 2x}{e^{3x}\sin x} = \lim\limits_{x\to 0} \dfrac{e^{-\frac{1}{x^2}} + 2x}{\sin x}$

$= \lim\limits_{x\to 0} \dfrac{e^{-\frac{1}{x^2}}}{x \cdot \frac{\sin x}{x}} + \dfrac{2}{\frac{\sin x}{x}} = 2 + \lim\limits_{x\to 0} \dfrac{e^{-\frac{1}{x^2}}}{x}$

$= 2 + \lim\limits_{x\to 0} \dfrac{\frac{1}{x}}{\frac{1}{e^{x^2}}} = 2 + \lim\limits_{x\to 0} \dfrac{\frac{-\frac{2}{x^3}}{x^2}}{-\frac{2}{x^3}e^{\frac{1}{x^2}}}$

$= 2 + \lim\limits_{x\to 0} \dfrac{x}{2e^{\frac{1}{x^2}}} = 2 + 0 = 2$

**14. (c) :**

| Value ($x_i$) | $f_i$ | c.f. | $f_i x_i$ |
|---|---|---|---|
| 4 | 5 | 5 | 20 |
| 5 | $f_1$ | $5 + f_1$ | 20 |
| 6 | 1 | $6 + f_1$ | 6 |
| 8 | $f_2$ | $6 + f_1 + f_2$ | 24 |
| 9 | 2 | $8 + f_1 + f_2$ | 18 |
| 11 | 3 | $11 + f_1 + f_2$ | 33 |
| 12 | 1 | $12 + f_1 + f_2$ | 12 |
| | $12 + f_1 + f_2$ | | $\Sigma f_i x_i = 133$ |

Sum of frequencies is $19 \Rightarrow f_1 + f_2 = 7$

Median is 6. So, median $= \left(\frac{19+1}{2}\right)^{\text{th}}$ term *i.e.*, $10^{\text{th}}$ term is 6

$\Rightarrow 6 + f_1 = 10 \Rightarrow f_1 = 4, f_2 = 3$

$$\text{Mean} = \frac{\sum x_i f_i}{\sum f_i} = \frac{4\times5 + 5\times4 + 6\times1 + 8\times3 + 9\times2 + 11\times3 + 12\times1}{19}$$

$$= \frac{133}{19} = 7 = \bar{x}$$

$$\text{Mean deviation about mean} = \alpha = \frac{\sum f_i |x_i - \bar{x}|}{N}$$

$$= \frac{5\times3 + 4\times2 + 1\times1 + 1\times3 + 2\times2 + 4\times3 + 5\times1}{19} = \frac{48}{19}$$

$$\text{Mean deviation about median} = \beta = \frac{\sum f_i |x_i - 6|}{N}$$

$$= \frac{5\times2 + 4\times1 + 1\times0 + 3\times2 + 2\times3 + 3\times5 + 1\times6}{19} = \frac{47}{19}$$

$$\sigma^2 = \frac{\sum f_i x_1^2}{N} - (\bar{x})^2 = \frac{5\times16 + 4\times25 + 1\times36 + 3\times64 + 2\times81 + 3\times121 + 1\times144}{19} - 49$$

$$= \frac{80 + 100 + 36 + 192 + 162 + 363 + 144 - 931}{19} = \frac{146}{19}$$

Now, **(P)** $7f_1 + 9f_2 = 7\times4 + 9\times3 = 28 + 27 = 55$

**(Q)** $19\alpha = 19\times\frac{48}{19} = 48$

**(R)** $19\beta = 19\times\frac{47}{19} = 47$

**(S)** $19\sigma^2 = 19\times\frac{146}{19} = 146$

**15. (b) :** **(P)** Let of $P(x) = 10x^3 - 45x^2 + 60x + 35$

$P'(x) = 30x^2 - 90x + 60 = 30(x-1)(x-2)$

$\therefore$ If $x \in [1, 2]$, $P(x)$ is monotonically decreasing

$P(1) = 10 - 45 + 60 + 35 = 60$

$P(2) = 80 - 180 + 120 + 35 = 55$

For $f(x)$ to be continuous, $f(1) = f(2) \Rightarrow \left[\frac{60}{n}\right] > \left[\frac{55}{n}\right]$

$\therefore \quad n \geq 9$

**(Q)** $x^3 + 3x$ is always increasing

(As $3(x^2+1) > 0 \ \forall \ x \in \mathbb{R}$)

$\therefore \quad 2n^2 - 13n - 15 > 0$, for $g(x)$ to be increasing

$\Rightarrow (2n - 15)(n + 1) > 0$

$n < -1, \ n > \frac{15}{2} \Rightarrow n > 7.5$

$\therefore \quad n_{\min} = 8 \qquad [\because \ n \in N]$

**(R)** $h(x) = (x-3)^n (x+3)^n (x^2 + 2x + 3)$

$h(x)$ is an even degree polynomial and hence for minima at $x = 3$, where both $x + 3$ and $x^2 + 2x + 3$ are increasing and positive, $(x - 3)$ must have an even power.

Since, $n > 5 \Rightarrow n_{\min} = 6 \qquad [\because \ n \in N]$

**(S)** Since, $\sin|x-k|$ is not differentiable at $x = k$ but $\cos\left|x - k + \frac{1}{2}\right|$ is differentiable as cosine functions is even function, so $l(x)$ is not differentiable at $x_0 \in \{0, 1, 2, 3, 4\}$ *i.e.*, required number is 5.

**16. (a) :** $\vec{u} \times \vec{v} = \vec{w}$ and $\vec{v} \times \vec{w} = \vec{u}$

$\Rightarrow \ \vec{u}, \vec{v}, \vec{w}$ are mutually orthogonal

as $\vec{u} + \vec{v}, \vec{u} \perp \vec{w}$ and $\vec{v} \perp \vec{w}$

$\therefore \quad |u||v| = |w|$ and $|v|\,|w| = |u|$

$\Rightarrow \ |u|\,|v|^2\,|w| = |w|\,|u| \Rightarrow |v|^2 = 1$

Now, $|\vec{u}||\vec{v}| = |\vec{w}|$

$\Rightarrow \ |u| = |w| = \sqrt{6} \ \Rightarrow \ \alpha^2 + \beta^2 + \gamma^2 = 6$ ...(i)

Now, $u \cdot w = 0 \ \Rightarrow \ \alpha + \beta - 2\gamma = 2 \ \Rightarrow \ \gamma = \dfrac{\alpha+\beta}{2}$ ...(ii)

and $-t\alpha + \beta + \gamma = 0,\ \alpha - t\beta + \gamma = 0,\ \alpha + \beta - t\gamma = 0$, then

$$\begin{vmatrix} -t & 1 & 1 \\ 1 & -t & 1 \\ 1 & 1 & -t \end{vmatrix} = 0 \Rightarrow -t^3 + 2 + 3t = 0$$

$\Rightarrow \ t^3 - 3t - 2 = 0 \ \Rightarrow \ (t+1)(t^2 - t - 2) = 0$

$\Rightarrow \ (t+1)^2 (t-2) = 0 \ \Rightarrow \ t = -1, 2$

**Case I :** When $t = -1$

$\alpha + \beta + \gamma = 0,\ \gamma = \dfrac{\alpha+\beta}{2} \Rightarrow \gamma = 0$ (From ii)

$\Rightarrow \ \alpha = -\beta$

From (1), $2\alpha^2 = 6 \Rightarrow \ \alpha = \pm\sqrt{3} \ \therefore \ \alpha = \sqrt{3}$ and $\gamma^2 = 0$

$\therefore \ \ (\beta + \gamma)^2 = 3$

**Case II :** When $t = 2$

$\gamma = \dfrac{\alpha+\beta}{2}, \beta = \dfrac{\alpha+\gamma}{2}, \ \alpha = \dfrac{\beta+\gamma}{2}$

$\Rightarrow \ \alpha = \beta = \gamma$

From (1), $3\alpha^2 = 6 \ \Rightarrow \ \alpha = \pm\sqrt{2} \ \Rightarrow \ \alpha = \sqrt{2} \ \therefore \ t + 3 = 5$

## PAPER-II

**1. (c) :** Given, $e^{x_0} + x_0 = 0$

and $g(x) = \dfrac{3xe^x + 3x - \alpha e^x - \alpha x}{3(e^x+1)}$

For $\alpha = 2$, $g(x) = \dfrac{3x(e^x+1) - 2(e^x+x)}{3(e^x+1)}$

$$= \frac{3x(e^x+1)}{3(e^x+1)} - \frac{2(e^x+x)}{3(e^x+1)} = x - \frac{2(e^x+x)}{3(e^x+1)}$$

$$\text{Let } L = \lim_{x\to x_0}\left|\frac{g(x)+e^{x_0}}{x-x_0}\right| = \lim_{x\to x_0}\left|\frac{x - \dfrac{2(e^x+x)}{3(e^x+1)} - x_0}{x-x_0}\right|$$

$\left[\because \ e^{x_0} = -x_0 \text{ (given)}\right]$

$$= \lim_{x\to x_0}\left|\frac{x - x_0 - \dfrac{2(e^x+x)}{3(e^x+1)}}{x-x_0}\right| = \lim_{x\to x_0}\left|1 - \frac{\dfrac{2(e^x+x)}{3(e^x+1)}}{x-x_0}\right|$$

$$= \lim_{x\to x_0}\left|1 - \frac{2}{3}\left\{\frac{(e^x+1)(e^x+1) - (e^x+x)(e^x)}{(e^x+1)^2}\right\}\right|$$

$$= \lim_{x\to x_0}\left|1 - \frac{2}{3}\left\{1 - \frac{(e^x+x)e^x}{(e^x+1)^2}\right\}\right| = \left|1 - \frac{2}{3}(1-0)\right| = \left|1 - \frac{2}{3}\right| = \frac{1}{3}$$

For $\alpha = 3$, $g(x) = \dfrac{3x(e^x+1) - 3(e^x+x)}{3(e^x+1)} = x - \dfrac{e^x+x}{e^x+1}$

$$\text{Let } M = \lim_{x\to x_0}\left|\frac{g(x)+e^{x_0}}{x-x_0}\right| = \lim_{x\to x_0}\left|\frac{x - \left(\dfrac{e^x+x}{e^x+1}\right) + e^{x_0}}{x-x_0}\right|$$

$$= \lim_{x\to x_0}\left|\frac{(x-x_0)}{x-x_0} - \frac{\left(\dfrac{e^x+x}{e^x+1}\right)}{x-x_0}\right| = \lim_{x\to x_0}\left|1 - \frac{\left(\dfrac{e^x+x}{e^x+1}\right)}{x-x_0}\right| = 0$$

**2. (b) :** Let $5x - 4y - 1 = 0$ ...(i)

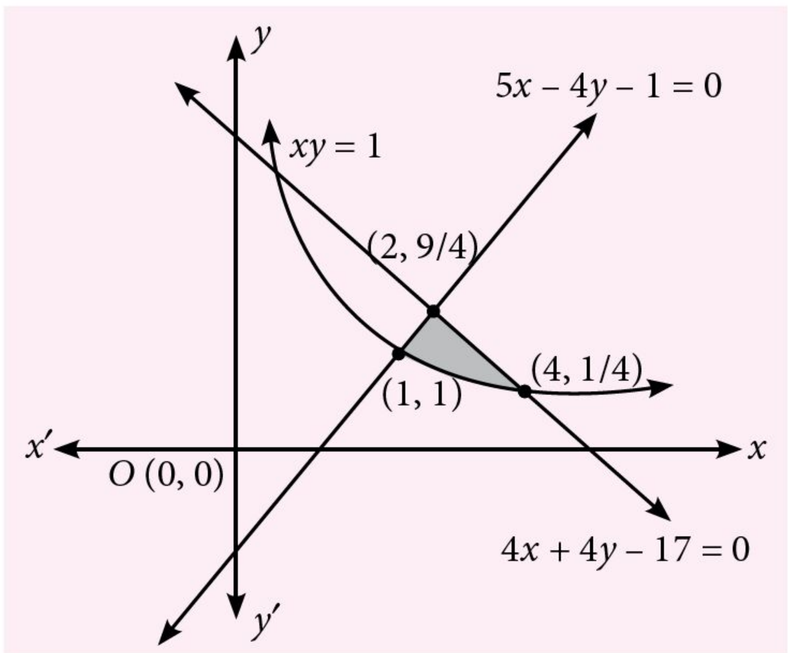
$4x + 4y - 17 = 0$ ...(ii)

$xy = 1$ ...(iii)

Intersection point of (i) & (ii) is $\left(2, \dfrac{9}{4}\right)$

Intersection point of (ii) & (iii) is $\left(4, \dfrac{1}{4}\right)$

Intersection point of (i) & (iii) is (1, 1)




$\therefore$ Required area

$$= \int_1^2 \left(\frac{5x-1}{4} - \frac{1}{x}\right)dx + \int_2^4 \left(\frac{17-4x}{4} - \frac{1}{x}\right)dx$$

$$= \frac{5}{4}\left(\frac{x^2}{2}\right)_1^2 - \frac{1}{4}(x)_1^2 - (\log_e x)_1^2 + \frac{17}{4}(x)_2^4 - \left(\frac{x^2}{2}\right)_2^4 - \left(\log_e(x)\right)_2^4$$

$$= \frac{5}{4}\left(\frac{3}{2}\right) - \frac{1}{4} - \log_e 2 + \log_e 1 + \frac{17}{4}(2) - 6 - \log_e 4 + \log_e 2$$

$$= \frac{15}{8} + \frac{33}{4} - 6 - \log_e 4 = \frac{33}{8} - \log_e 4 \text{ sq. units}$$

**3. (c) :** Clearly, $\theta = 0$ satisfy the given equation. So, it is a solution.

Given, $\theta = \tan^{-1}(2\tan\theta) - \frac{1}{2}\sin^{-1}\left(\frac{6\tan\theta}{9+\tan^2\theta}\right)$

Let $\frac{\tan\theta}{3} = \alpha \Rightarrow \tan\theta = 3\alpha \Rightarrow \theta = \tan^{-1}(3\alpha)$

So, $\tan^{-1}(3\alpha) = \tan^{-1}(6\alpha) - \frac{1}{2}\sin^{-1}\left(\frac{18\alpha}{9+9\alpha^2}\right)$

$\Rightarrow \tan^{-1}(3\alpha) = \tan^{-1}(6\alpha) - \frac{1}{2}\sin^{-1}\left(\frac{2\alpha}{1+\alpha^2}\right)$

$\Rightarrow \frac{1}{2}\sin^{-1}\left(\frac{2\alpha}{1+\alpha^2}\right) = \tan^{-1}(6\alpha) - \tan^{-1}(3\alpha)$

$\Rightarrow \sin^{-1}\left(\frac{2\alpha}{1+\alpha^2}\right) = 2\left\{\tan^{-1}\left(\frac{6\alpha-3\alpha}{1+18\alpha^2}\right)\right\}$

$\Rightarrow \sin^{-1}\left(\frac{2\alpha}{1+\alpha^2}\right) = 2\tan^{-1}\left(\frac{3\alpha}{1+18\alpha^2}\right)$

$\Rightarrow \tan^{-1}\left(\frac{2\alpha}{1-\alpha^2}\right) - \tan^{-1}\left(\frac{3\alpha}{1+18\alpha^2}\right) = \tan^{-1}\left(\frac{3\alpha}{1+18\alpha^2}\right)$

$\Rightarrow \tan^{-1}\left[\frac{\frac{2\alpha}{1-\alpha^2} - \frac{3\alpha}{1+18\alpha^2}}{1+\left(\frac{2\alpha}{1-\alpha^2}\right)\left(\frac{3\alpha}{1+18\alpha^2}\right)}\right] = \tan^{-1}\left(\frac{3\alpha}{1+18\alpha^2}\right)$

$\Rightarrow \frac{39\alpha^2-1}{23\alpha^2-18\alpha^4+1} = \frac{3}{1+18\alpha^2}$

Put $\alpha^2 = t$,

$\frac{39t-1}{23t-18t^2+1} = \frac{3}{1+18t} \Rightarrow 756t^2 - 48t - 4 = 0$

Here, product of roots is negative it means one root is positive and other root is negative but negative root is neglected as $\alpha^2 = t$.

Thus, only two real solution exists.

Hence, total number of solution is 3.

**4. (a) :** Given, $4x - 3y = 12\alpha$ ...(i)

$4x + 3y = 12/\alpha$ ...(ii)

On multiplying (i) & (ii), we get

$16x^2 - 9y^2 = 144 \Rightarrow \frac{x^2}{9} - \frac{y^2}{16} = 1$

Equation of tangent of hyperbola in slope form is

$y = mx \pm \sqrt{a^2m^2 - b^2}$

$m = \frac{4\sqrt{2}}{3}$ $\left[\text{As parallel to } 4x - \frac{3}{\sqrt{2}}y = 0\right]$

$y = \frac{4\sqrt{2}}{3}x \pm \sqrt{9\times\frac{32}{9} - 16} = \frac{4\sqrt{2}}{3}x \pm 4$

$\Rightarrow y = \frac{4\sqrt{2}}{3}x + 4$ [As $(0, q)$, $q > 0$]

Since, it passes through $(p, 0)$ and $(0, q)$, then

$p = \frac{-3}{\sqrt{2}}$ and $q = 4$

$\therefore\ pq = \frac{-3}{\sqrt{2}} \times 4 = -6\sqrt{2}$

**5. (a, b) :** We have, $P = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$ and $Q = \begin{bmatrix} x & y \\ z & 4 \end{bmatrix}$

Let $R = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Now, $QR = RP$

$\Rightarrow \begin{bmatrix} x & y \\ z & 4 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} ax+cy & bx+dy \\ az+4c & bz+4d \end{bmatrix} = \begin{bmatrix} 2a & 3b \\ 2c & 3d \end{bmatrix}$

$\Rightarrow$ $ax + cy = 2a$, $az + 4c = 2c$ and
$bx + dy = 3b$, $bz + 4d = 3d$

$\Rightarrow \frac{x-2}{-y} = \frac{c}{a} = \frac{z}{-2}$ and $\frac{x-3}{-y} = \frac{d}{b} = \frac{z}{-1}$

$\Rightarrow -2x + 4 = -yz$ and $-x + 3 = -yz$

$\Rightarrow -2x + 4 = -x + 3 \Rightarrow x = 1$

$\Rightarrow yz = x - 3 = 1 - 3 = -2$

Now, $Q - 2I = \begin{bmatrix} x & y \\ z & 4 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} x-2 & y \\ z & 2 \end{bmatrix}$

$\Rightarrow |Q - 2I| = 2x - 4 - yz = 2 - 4 + 2 = 0$

Similarly, $|Q-3I| = \begin{vmatrix} x-3 & y \\ z & 1 \end{vmatrix} = x - 3 - yz = 1 - 3 + 2 = 0$

$|Q-6I| = \begin{vmatrix} x-6 & y \\ z & -2 \end{vmatrix} = -2x + 12 - yz = -2 + 12 + 2 = 12$

**6. (a, c) :** Equation of chord of a parabola with mid-point $(x_1, y_1)$ is $T = S_1$

Let mid-point of chord be $(h, k)$

$\Rightarrow ky - \left(\frac{x+h}{2}\right) = k^2 - h \Rightarrow x - 2ky + 2k^2 - h = 0$

For intersecting point of chord and parabola,

$y^2 - 2ky + 2k^2 - h = 0 \Rightarrow (y - k)^2 = h - k^2$

Let $y_1$ and $y_2$ are the intersecting ordinates of chord and parabola.

$|y_1 - y_2| = \sqrt{4k^2 - 4(2k^2 - h)} = \sqrt{4h - 4k^2} = 2\sqrt{h - k^2}$

$y_1 + y_2 = 2k$ and $y_1 y_2 = 2k^2 - h$

Area between parabola and chord $= \frac{4}{3}$

$$\Rightarrow \int_{y_1}^{y_2} [(2ky - 2k^2 + h) - y^2]\, dy = \frac{4}{3}$$

$$\Rightarrow \left[ ky^2 + (h - 2k^2)y - \frac{y^3}{3} \right]_{y_1}^{y_2} = \frac{4}{3}$$

$$\Rightarrow k(y_2^2 - y_1^2) + (h - 2k^2)(y_2 - y_1) - \frac{1}{3}(y_2^3 - y_1^3) = \frac{4}{3}$$

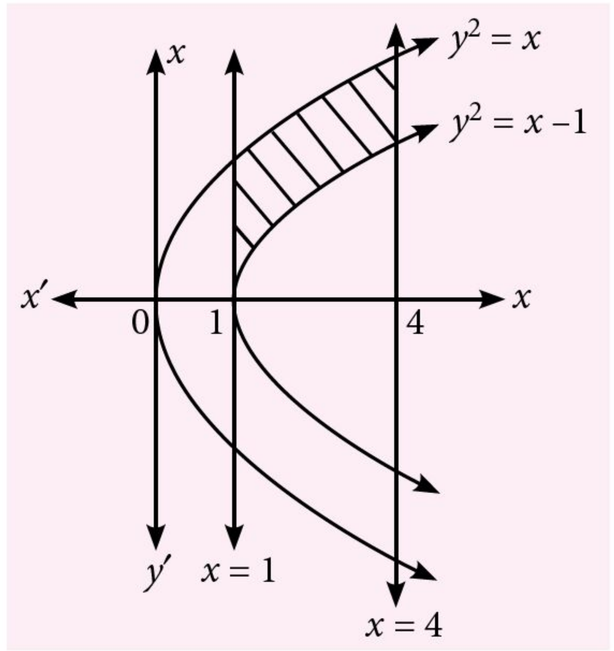
$$\Rightarrow (y_2 - y_1)\left[ \begin{matrix} k(y_1 + y_2) + (h - 2k^2) \\ -\frac{1}{3}(y_1^2 + y_2^2 + y_1 y_2) \end{matrix} \right] = \frac{4}{3}$$

$$\Rightarrow 2\sqrt{h - k^2}\left[ k(2k) + h - 2k^2 - \frac{1}{3}(h + 2k^2) \right] = \frac{4}{3}$$

$\Rightarrow (h - k^2)^{3/2} = 1 \Rightarrow x - y^2 = 1 \Rightarrow y^2 = x - 1$

$$\text{Area} = \int_1^4 \left( \sqrt{x} - \sqrt{x-1} \right) dx = \frac{2}{3}\left[ x^{3/2} - (x-1)^{3/2} \right]_1^4$$

$$= \frac{2}{3}\left[ 8 - 3\sqrt{3} - 1 \right] = \frac{14}{3} - 2\sqrt{3} \text{ sq.units}$$




Since, $(4, \sqrt{3})$ satisfy $y^2 = x - 1$, then $(4, \sqrt{3}) \in S$.

**7. (a,c)**

**8. (b,c,d)**

**9. (0.75) :** We have, $x^2 \frac{dy}{dx} + xy = x^2 + y^2$

Put $y = vx$

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

Now, $x^2\left( v + x\frac{dv}{dx} \right) + x(vx) = x^2 + (vx)^2$

$$\Rightarrow x^2\left( v + x\frac{dv}{dx} \right) + vx^2 = x^2(1 + v^2)$$

$$\Rightarrow v + x\frac{dv}{dx} + v = (1 + v^2) \Rightarrow x\frac{dv}{dx} = 1 + v^2 - 2v$$

$$\Rightarrow \int \frac{dv}{1 + v^2 - 2v} = \int \frac{dx}{x} \Rightarrow \int \frac{1}{(v-1)^2} = \int \frac{dx}{x}$$

$$\Rightarrow -\frac{1}{v-1} = \log|x| + c$$

$$\Rightarrow \frac{x}{x - y} = \log x + c \qquad \left( \because\ x > \frac{1}{e} \right)$$

Given, $y(1) = 0$

$$\Rightarrow \frac{1}{1 - 0} = \log(1) + c \Rightarrow c = 1$$

So, $\frac{x}{x - y} = \log x + 1 \Rightarrow \frac{x}{x - y} = \log(ex)$

Now, $y(e) = \frac{e}{2}$ and $y(e^2) = \frac{2e^2}{3}$

$$\therefore \quad 2\frac{(y(e))^2}{y(e^2)} = \frac{2\left( \frac{e^2}{4} \right)}{2\left( \frac{e^2}{3} \right)} = \frac{3}{4} = 0.75$$

**10. (6) :** Given, $\left( 1 + \frac{2x}{5} \right)^{23} = \sum_{i=0}^{23} a_i x^i$

For $\left( 1 + \frac{2x}{5} \right)^{23}$, the numerically greatest term is $\frac{n+1}{\left| \frac{5}{2x} \right| + 1}$

Putting $x = 1$ and $n = 23$, we get

$$\therefore \quad \frac{n+1}{\frac{5}{2x} + 1} = \frac{24}{\frac{5}{2} + 1} = \frac{48}{7} \Rightarrow \left[ \frac{48}{7} \right] = 6$$

(where $[\cdot]$ greatest integer function)

So, $T_7$ is numerically greatest term.

Hence $r = 6$.

**11. (0.3) :** Let $A_1$ : Bulb is produced by unit $M_1$.

$A_2$ : Bulb is produced by unit $M_2$.

$A_3$ : Bulb is produced by unit $M_3$.

$E$ : Bulb produced is defective.

$$P(A_1) = \frac{2}{5},\ P(A_2) = \frac{2}{5}, P(A_3) = \frac{1}{5},\ P(E) = \frac{20}{100}$$

$$P(E/A_1) = \frac{15}{100} = \frac{3}{20}, P\left( A_2 / E \right) = \frac{2}{5}$$

Now, $P(E) = P(E/A_1) \cdot P(A_1) + P(E/A_2) \cdot P(A_2) + P(E/A_3) \cdot P(A_3)$

$$\Rightarrow \frac{20}{100}=\frac{3}{20}\cdot\frac{2}{5}+P(E/A_2).\frac{2}{5}+P(E/A_3).\frac{1}{5}$$

$$\Rightarrow \frac{1}{5}=\frac{3}{50}+P(E/A_2)\cdot\frac{2}{5}+\frac{1}{5}\cdot P(E/A_3)$$

$$\Rightarrow 2P(E/A_2)+P(E/A_3)=\frac{7}{10} \quad \ldots(i)$$

Now, $P(A_2/E)=\frac{P(E/A_2)}{P(E)}P(A_2)$

$$\Rightarrow \frac{2}{5}=\frac{P(E/A_2)\cdot\frac{2}{5}}{\frac{1}{5}} \Rightarrow P(E/A_2)=\frac{1}{5} \quad \ldots(ii)$$

From (i) and (ii), we get

$$P(E/A_3)=\frac{3}{10}$$

$\therefore$ Required probability, $P(E/A_3)=\frac{3}{10}=0.3$

**12. (–2) :** Given, $\vec{X}, \vec{Y}$ and $\vec{Z}$ lies in a plane.

$$\left[\vec{X} \quad \vec{Y} \quad \vec{Z}\right]=0$$

As, $\left[\vec{X} \quad \vec{Y} \quad \vec{Z}\right]=\begin{vmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 3 & 1 & 2\end{vmatrix}\neq 0$

$$\Rightarrow \begin{vmatrix}\alpha & \beta & -1\\ -1 & \alpha & \beta\\ \beta & -1 & \alpha\end{vmatrix}=\alpha^3+\beta^3+(-1)^3-3\alpha\beta\ (-1)=0$$

$\Rightarrow \alpha+\beta-1=0$

So, $\alpha+\beta-3=-2$

**13. (–2) :** We have, $\alpha=\arg\left(\sum_{n=1}^{2025}(-\omega)^n\right)$

$=\arg(-\omega+\omega^2-\omega^3+\ldots+(-\omega)^{2025})$

$$=\arg\left(\frac{-\omega\left((-\omega)^{2025}-1\right)}{-\omega-1}\right)=\arg\left(\frac{-\omega\left((-\omega)^{3\times675}-1\right)}{-\omega-1}\right)$$

$$=\arg\left(\frac{-\omega(-1-1)}{-\omega-1}\right)=\arg\left(\frac{2\omega}{\omega^2}\right)=\arg\left(\frac{2}{\omega}\right)=\arg(2\omega^2)$$

$$=\tan^{-1}\left(\frac{-\sqrt{3}}{-1}\right)=-\pi+\frac{\pi}{3}=\frac{-2\pi}{3}$$

[$\because$ $x<0$ and $y<0$, then $\arg(z)=-\pi+\theta$]

$$\Rightarrow \frac{3\alpha}{\pi}=-2$$

**14. (0.25) :** We have, $f:\mathbb{R}\to\mathbb{R}$,

$f(x)=\log_e(x^2+2x+4)=\log_e((x+1)^2+3)$

and $g:\mathbb{R}\to(0,4)$, $g(x)=\frac{4}{1+e^{-2x}}=y$ (let)

$$\Rightarrow 1+e^{-2x}=\frac{4}{y} \Rightarrow -2x=\log\left(\frac{4}{y}-1\right)$$

$$\Rightarrow x=-\frac{1}{2}\log\left(\frac{4-y}{y}\right)=\frac{1}{2}\log\left(\frac{y}{4-y}\right)$$

$$\Rightarrow g^{-1}(x)=\frac{1}{2}\log\left(\frac{x}{4-x}\right)$$

Hence, $g^{-1}(2)=0$

Now, $\frac{d}{dx}\left(f\left(g^{-1}(x)\right)\right)=\frac{d}{dx}\left(\log\left((g^{-1}(x)+1)^2+3\right)\right)$

$$=\frac{1}{\left(g^{-1}(x)+1\right)^2+3}\times2(g^{-1}(x)+1)\times\frac{d}{dx}g^{-1}(x)$$

$$=\frac{2\left(g^{-1}(x)+1\right)}{\left(g^{-1}(x)+1\right)^2+3}\times\frac{d}{dx}\left(\frac{1}{2}\log\left(\frac{x}{4-x}\right)\right)$$

$$=\frac{2\left(g^{-1}(x)+1\right)}{\left(g^{-1}(x)+1\right)^2+3}\times\frac{1}{2}\times\frac{4-x}{x}\times\frac{4-x+x}{(4-x)^2}$$

$$=\frac{2\left(g^{-1}(x)+1\right)}{\left(g^{-1}(x)+1\right)^2+3}\times\frac{2}{x(4-x)}$$

$$\left.\frac{d}{dx}f(g^{-1}(x))\right|_{x=2}=\frac{2\left(g^{-1}(2)+1\right)}{\left(g^{-1}(2)+1\right)^2+3}\times\frac{2}{2(4-2)}$$

$$=\frac{2\times2}{(1+3)\times2\times2}=\frac{1}{4}=0.25$$

**15. (3) :** We have,

$$\alpha=\frac{1}{\sin60^\circ\sin61^\circ}+\frac{1}{\sin62^\circ\sin63^\circ}+\ldots+\frac{1}{\sin118^\circ\sin119^\circ}$$

$$\Rightarrow \sin1^\circ\cdot\alpha=\frac{\sin(61^\circ-60^\circ)}{\sin60^\circ\sin61^\circ}+\frac{\sin(63^\circ-62^\circ)}{\sin62^\circ\sin63^\circ}+\ldots+\frac{\sin(119^\circ-118^\circ)}{\sin118^\circ\sin119^\circ}$$

$$=\cot60^\circ-\cot61^\circ+\cot62^\circ-\cot63^\circ+\ldots+\cot118^\circ-\cot119^\circ$$

$$=\cot60^\circ-\cot61^\circ+\cot62^\circ-\cot63^\circ+\ldots+\cot89^\circ-\cot90^\circ-\cot89^\circ+\ldots-\cot62^\circ+\cot61^\circ$$

$$=\cot60^\circ$$

$$\Rightarrow \alpha=\frac{1}{\sqrt{3}\sin1^\circ}=\frac{\operatorname{cosec}1^\circ}{\sqrt{3}} \quad \therefore \quad \left(\frac{\operatorname{cosec}1^\circ}{\alpha}\right)^2=3$$

**16. (21)**

SOLVED PAPER **2025**

Exam Held on 23rd April

# KEAM ENGINEERING

**1.** Let $A, B, C$ be any three finite sets.
If $n(A \times B) = 160$, $n(B \times C) = 80$ and $n(C \times A) = 200$, then $n(A) =$
(a) 10 (b) 18 (c) 16 (d) 12
(e) 20

**2.** Let $f(x) = x^2 - 10x - 19$, $x \in \mathbb{R}$. Then the inverse image of 5, $f^{-1}(5) =$
(a) $\{-2, -12\}$ (b) $\{-2, 12\}$
(c) $\{2, -12\}$ (d) $\{2, 12\}$
(e) $\phi$

**3.** Let $f(x) = \cos x$. Then the value of
$\frac{1}{2}[f(x+y) + f(y-x)] - f(x)f(y)$ is equal to
(a) 2 (b) $-2$ (c) 1 (d) $-1$
(e) 0

**4.** Let $f(x) = \log_5 x (x > 0)$ and $g(x) = \cos^{-1} x (-1 \le x \le 1)$. Then the domain of $gof$ is
(a) $(0, 1]$ (b) $[-1, \alpha)$
(c) $[0, \alpha)$ (d) $\left[\frac{1}{5}, 5\right]$
(e) $[-1, 5]$

**5.** Let $z = 1 + \frac{1}{i}$. Then the value of $z^4$ is equal to
(a) 4 (b) $-4$ (c) $1 - i$ (d) $1 + i$
(e) $i$

**6.** The modulus of the complex number $\left(2\sqrt{2} + i2\sqrt{2}\right)^2$ is equal to
(a) 64 (b) 4 (c) 32 (d) 8
(e) 16

**7.** If $z + \overline{z} = 6$ and $z - \overline{z} = 4i$, then $|z|^2 =$
(a) 36 (b) 16 (c) 15 (d) 13
(e) 9

**8.** Let $z = \frac{2-i}{\alpha+i}$, where $\alpha$ is a real number.
If $4\,\mathrm{Re}(z) = 3\,\mathrm{Im}(\overline{z})$, then the value of $\alpha$ is
(a) 5 (b) $-5$ (c) 3 (d) 2
(e) $-2$

**9.** In a G.P., the first and third terms are 4 and 8 respectively. Then the 21st term is
(a) 4012 (b) 4064 (c) 4098 (d) 2048
(e) 4096

**10.** Let $a_1, a_2, a_3, \ldots$ be in G.P. If $a_1 \cdot a_2 \cdot a_3 = 64$ and $a_1 \cdot a_2 \cdot a_3 a_4 \cdot a_5 = 32$, then common ratio is
(a) $\frac{1}{3}$ (b) $\frac{1}{8}$ (c) $\frac{1}{6}$ (d) $\frac{1}{2}$
(e) $\frac{1}{4}$

**11.** The general term of a sequence is $t_n = \frac{n(n+6)}{n+4}$, $n = 1, 2, 3, \ldots$. If $t_n = 5$, then the value of $n$ is
(a) 2 (b) 3 (c) 4 (d) 5
(e) 6

**12.** The product of first 5 terms of a G.P., whose terms are increasing, is 32. The third term of the G.P. is
(a) 2 (b) 1/2 (c) 4 (d) 1/8
(e) 8

**13.** Let $\alpha = \sum_{k=0}^{5} {}^{10}C_{2k}$ and $\beta = \sum_{k=0}^{4} {}^{10}C_{2k+1}$.
Then $\alpha - \beta$ is equal to
(a) 32 (b) 64 (c) 128 (d) 256
(e) 0

**14.** If $\alpha = {}^nC_r$ and $\beta = {}^nC_{r-1}$, then $1 + \frac{\alpha}{\beta}$ is equal to
(a) $\frac{n+1}{r-1}$ (b) $\frac{n+1}{r}$
(c) $\frac{n-1}{1}$ (d) $\frac{n-r+1}{r}$
(e) $\frac{n+1}{r+1}$

**15.** If ${}^{11}P_r = 7920$, then the value of $r$ is equal to
(a) 7 (b) 6 (c) 5 (d) 4
(e) 3

**16.** In the binomial expansion of $(2x + \alpha)^8$, the co-efficients of $x^2$ and $x^3$ are equal. Then the value of $\alpha$ is equal to

(a) 2 (b) $\frac{1}{4}$ (c) 4 (d) $\frac{1}{2}$
(e) 3

**17.** Let $A = \{0, 2, 4, 6, 8\}$. The number of 5-digit numbers that can be formed using the digits in $A$ without replacement, is

(a) 120 (b) 96 (c) 88 (d) 64
(e) 32

**18.** Let $A$ be a $3 \times 3$ matrix and let $B = 3A$. If $|A| = 5$, then the value of $\frac{|\text{adj } B|}{|3A|}$ is equal to

(a) 27 (b) 125 (c) 25 (d) 135
(e) 81

**19.** If $\begin{pmatrix} -1 & 2 \\ 3 & -4 \\ -5 & 6 \end{pmatrix}\begin{pmatrix} 7 \\ 8 \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \\ 13 \end{pmatrix}$, then the value of $\alpha + \beta$ is equal to

(a) −18 (b) 18 (c) 21 (d) −21
(e) −2

**20.** If the matrix $\begin{bmatrix} 8-k & 2 \\ -2 & 4-k \end{bmatrix}$ is singular, then the value of $k$ is equal to

(a) 6 (b) 5 (c) 4 (d) 3
(e) 2

**21.** The following system of equations
$x + y + z = 1$
$2x + 3y - mz = 2$
$3x + 5y + 3z = 3$
has no unique solution. Then the value of $m$ is equal to

(a) 3 (b) 5 (c) 2 (d) −2
(e) −3

**22.** The set of all $x$ satisfying the inequalities $-4 \le 2 - 3x < 7$ is

(a) $\left(2, \frac{5}{3}\right)$ (b) $\left[2, \frac{5}{3}\right)$
(c) $\left[\frac{-11}{3}, 2\right]$ (d) $\left(\frac{-5}{3}, 2\right]$
(e) $\left[\frac{-7}{3}, 2\right)$

**23.** $-5 < x \le -1$ implies $-21 < 5x + 4 \le b$, the least value of $b$ is

(a) 5 (b) −5 (c) −4 (d) 4
(e) −1

**24.** $\tan 15° + \tan 75° =$

(a) $\sqrt{5}+1$ (b) 2 (c) $\sqrt{7}-1$ (d) 4
(e) 0

**25.** If $x + z = 2y$ and $y = \frac{\pi}{4}$, then $\tan x \tan y \tan z =$

(a) 1 (b) $\tan(x - y)$
(c) $\tan(z - y)$ (d) $\frac{1}{2}$
(e) 0

**26.** If $\sin x + \sin y = a$, $\cos x + \cos y = b$ and $x + y = \frac{2\pi}{3}$, then the value of $\frac{a}{b}$ is equal to

(a) $\frac{\sqrt{3}}{3}$ (b) $2\sqrt{3}$ (c) $\sqrt{3}$ (d) $4\sqrt{3}$
(e) $\frac{\sqrt{3}}{6}$

**27.** If $\sin\alpha = \frac{12}{13}$, where $\frac{\pi}{2} < \alpha < \frac{3\pi}{2}$ then the value of $\tan\alpha$ is equal to

(a) $\frac{5}{12}$ (b) $\frac{13}{5}$ (c) $\frac{-12}{5}$ (d) $\frac{-13}{5}$
(e) $\frac{-1}{12}$

**28.** If $f(x) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, then $f\left(\frac{1}{\sqrt{3}}\right)$ is equal to

(a) $\frac{\pi}{6}$ (b) $\frac{2\pi}{3}$ (c) $\frac{\pi}{3}$ (d) $\frac{4\pi}{3}$
(e) 0

**29.** If $5\sin^{-1}\alpha + 3\cos^{-1}\alpha = \pi$, then $\alpha$ is equal to

(a) $\frac{1}{\sqrt{2}}$ (b) 1 (c) $\frac{-1}{\sqrt{2}}$ (d) −1
(e) 0

**30.** If $\theta = \cot^{-1}\sqrt{\frac{1-x}{1+x}}$, then $\sec^2\theta =$

(a) $\frac{1+x}{2}$ (b) $\frac{1-x}{2}$ (c) $\frac{2}{1-x}$ (d) $x$
(e) $2x$

**31.** The straight line $ax + by + c = 0$ passes through the point $(-10, 7)$. If the line is perpendicular to $11x - 7y = 13$, then the value of $c$ is equal to

(a) 8 (b) −7 (c) 13 (d) −13
(e) 5

**32.** Let $ABC$ be an equilateral triangle. If the coordinates of $A$ are $(-2, 2)$ and the side $BC$ is along the line $x + y = 6$, then the length of the side of the triangle is

(a) $2\sqrt{3}$ (b) $3\sqrt{2}$ (c) $4\sqrt{6}$ (d) $6\sqrt{6}$
(e) $2\sqrt{6}$

**33.** The focus of the parabola $x^2 - 4x + 8y + 4 = 0$ is

(a) $(-2, -2)$ (b) $(1, 1)$ (c) $(2, 1)$ (d) $(2, -2)$
(e) $(1, 2)$

**34.** A circle touches the $x$-axis at $(9, 0)$. If it also touches the straight line $y = 14$, then the equation of the circle is

(a) $(x - 9)^2 + (y - 7)^2 = 49$
(b) $x^2 + (y - 7)^2 = 49$
(c) $(x - 9)^2 + y^2 = 49$
(d) $(x - 9)^2 + (y - 7)^2 = 81$
(e) $(x - 7)^2 + (y - 9)^2 = 49$

**35.** The length of major axis and minor axis of an ellipse are, respectively, $m$ and $n$. If $m^2 - n^2 = 45$ and the eccentricity of the ellipse is $\frac{\sqrt{5}}{3}$, then the length of the major axis is

(a) 13 (b) 6 (c) 12 (d) 18
(e) 9

**36.** The vertex of the parabola $4y = x^2 - 6x + 17$ is

(a) $(3, 2)$ (b) $(4, 3)$ (c) $(4, 2)$ (d) $(3, 7)$
(e) $(7, 2)$

**37.** The eccentricity of the hyperbola

$$\frac{(2x-6)^2}{2} - \frac{(4y+7)^2}{16} = 1 \text{ is}$$

(a) $\sqrt{5}$ (b) $\frac{\sqrt{5}}{2}$ (c) $\sqrt{3}$ (d) $\sqrt{10}$
(e) $\frac{\sqrt{3}}{2}$

**38.** Let $\vec{a}+\vec{b} = \lambda\hat{i}+16\hat{j}-18\hat{k}$ and $\vec{a}-\vec{b} = 2\hat{i}+8\hat{j}+\lambda\hat{k}$. If $\vec{a}+\vec{b}$ is perpendicular to $\vec{a}-\vec{b}$, then $|\vec{a}|=$

(a) $5\sqrt{13}$ (b) $\sqrt{174}$ (c) $\sqrt{184}$ (d) $13\sqrt{5}$
(e) $\sqrt{194}$

**39.** If $|\vec{a}|=12$ and the projection of $\vec{a}$ on $\vec{b}$ is $6\sqrt{3}$, then the angle between $\vec{a}$ and $\vec{b}$ is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{2\pi}{3}$
(e) $\frac{3\pi}{4}$

**40.** Let $\vec{a} = 6\hat{i}+2\hat{j}+3\hat{k}$. If $\vec{b}$ is parallel to $\vec{a}$ and $\vec{a}\cdot\vec{b} = \frac{49}{2}$, then $|\vec{b}|=$

(a) 49 (b) 7 (c) 14 (d) $7\sqrt{2}$
(e) $\frac{7}{2}$

**41.** If $|\vec{a}+\vec{b}|=\frac{\sqrt{14}}{2}$ where $\vec{a}$ and $\vec{b}$ are unit vectors, then the value of $|\vec{a}+\vec{b}|^2 - |\vec{a}-\vec{b}|^2$ is equal to

(a) 3 (b) 4 (c) $\sqrt{5}$ (d) $\sqrt{7}$
(e) 7

**42.** Let $\alpha$, $\beta$ and $\gamma$ be the angles made by a straight line with the $x$-axis, $y$-axis and $z$-axis respectively. If $\cos\alpha + \cos\beta + \cos\gamma = \frac{5}{3}$, then the value of $\cos\alpha\cos\beta + \cos\beta\cos\gamma + \cos\gamma\cos\alpha$ is equal to

(a) $\frac{11}{3}$ (b) $\frac{8}{9}$ (c) $\frac{11}{9}$ (d) $\frac{7}{3}$
(e) $\frac{7}{9}$

**43.** A straight line passing through $(6, 1, 3)$ meets the line $\frac{x-1}{2}=\frac{y}{1}=\frac{z-2}{3}$ at $Q$. If the lines are perpendicular to each other, then the coordinates of $Q$ are

(a) $(2,1,3)$ (b) $(1,2,3)$ (c) $(3,1,5)$ (d) $(2,-1,3)$
(e) $(-1, 2, 3)$

**44.** The angle between the lines $\frac{x-3}{1}=\frac{y+1}{-1}=\frac{z-2}{-1}$ and $\frac{x+1}{2}=\frac{y-2}{2}=\frac{z+3}{-2}$ is

(a) $\cos^{-1}\left(\frac{\sqrt{2}}{6}\right)$ (b) $\cos^{-1}\left(\frac{\sqrt{6}}{6}\right)$
(c) $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$ (d) $\cos^{-1}\left(\frac{1}{3}\right)$
(e) $\cos^{-1}\left(\frac{\sqrt{2}}{3}\right)$

**45.** A straight line passes through the points $(10, 8, 6)$ and $(13, 9, 4)$. A unit vector parallel to this line is

(a) $\frac{1}{\sqrt{17}}(3\hat{i}+2\hat{j}+2\hat{k})$ (b) $\frac{1}{\sqrt{6}}(\hat{i}+\hat{j}-2\hat{k})$
(c) $\frac{1}{\sqrt{14}}(3\hat{i}+\hat{j}+2\hat{k})$ (d) $\frac{1}{\sqrt{17}}(3\hat{i}+\hat{j}+2\hat{k})$
(e) $\frac{1}{\sqrt{14}}(3\hat{i}+\hat{j}-2\hat{k})$

**46.** A box contains 4 red and 6 white marbles. Two successive draws of 3 balls are made without replacement. The probability that in the first draw, all the 3 balls are white and in the second draw, all the 3 balls are red, is

(a) $\frac{2}{105}$ (b) $\frac{1}{70}$ (c) $\frac{4}{105}$ (d) $\frac{1}{105}$

(e) $\frac{1}{35}$

**47.** Let $A$ and $B$ be two events. If $P(A|B) = 0.4$, $P(A|B') = 0.7$ and $P(B) = 0.7$, then $P(A) =$

(a) 0.44 (b) 0.54 (c) 0.49 (d) 0.5

(e) 0.65

**48.** The standard deviation of the numbers –3, 0, 3, 8 is

(a) $\frac{\sqrt{60}}{2}$ (b) $\frac{\sqrt{62}}{2}$ (c) $\frac{\sqrt{65}}{2}$ (d) $\frac{\sqrt{66}}{2}$

(e) $\frac{\sqrt{67}}{2}$

**49.** An unbiased die is tossed until 5 appears. If $X$ denotes the number of tosses required, then

$\frac{P(X=2)}{P(X=5)} =$

(a) $\frac{25}{36}$ (b) $\frac{125}{216}$ (c) $\frac{216}{125}$ (d) $\frac{36}{25}$

(e) $\frac{216}{25}$

**50.** $\lim\limits_{x\to 0} \frac{x^2}{\sqrt{2}-\sqrt{1+\cos x}}$ is equal to

(a) $4\sqrt{2}$ (b) 4 (c) $2\sqrt{2}$ (d) $\sqrt{2}$

(e) 0

**51.** Let $f(x) = \begin{cases} \frac{\tan\alpha x + (\beta+1)\tan x}{x}, & \text{for } x \neq 0 \\ 5, & \text{for } x = 0 \end{cases}$ be continuous at $x = 0$. Then the value of $\alpha + \beta$ is equal to

(a) 2 (b) 3 (c) 4 (d) 5

(e) 6

**52.** The domain of the function $f(x) = \sqrt{x-3} + 4\sqrt{5-x}$ is

(a) [1, 2] (b) [2, 4] (c) [3, 5] (d) [3, 20]

(e) [12, 20]

**53.** If $f(x) = \frac{3^x}{3^x + \sqrt{3}}$, then $f(x) + f(1 - x)$ is equal to

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$ (c) $2\sqrt{3}$ (d) 1

(e) 0

**54.** $\lim\limits_{x\to 0} \frac{\sqrt{\cos^2 x+3} - \sqrt{\cos^2 x+\sin x+3}}{x} =$

(a) $\frac{1}{4}$ (b) $\frac{-1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{-1}{2}$

(e) –1

**55.** If $f(x) = |x^2 + x - 6|$ is not differentiable at $x = a$ and $x = b$, then $a^2 + b^2 =$

(a) 11 (b) 14 (c) 12 (d) 13

(e) 16

**56.** Let $f(x) = |\sin 3x| - |\cos 3x|$, where $\frac{\pi}{6} \le x \le \frac{\pi}{3}$. Then the value of $f'\left(\frac{\pi}{4}\right)$ is equal to

(a) $-3\sqrt{2}$ (b) $3\sqrt{2}$ (c) $\frac{-3}{\sqrt{2}}$ (d) $\frac{3}{\sqrt{2}}$

(e) 0

**57.** Let $h(x) = f\left(\sqrt{g(x)}\right)$. If $f'(3) = 6$, $g'(3) = 3$ and $g(3) = 9$, then the value of $h'(3)$ is equal to

(a) 1 (b) 3 (c) 6 (d) 9

(e) 18

**58.** Let $f(x) = (\cos^2 x)(a + \cos x)$. If $f'\left(\frac{\pi}{3}\right) = 0$, then the value of $a$ is equal to

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{3}{4}$ (c) $\frac{-3}{4}$ (d) $\frac{-3}{2}$

(e) –1

**59.** If $y = \tan^{-1}(x^2 - x)$, then $\frac{dy}{dx} =$

(a) $\frac{2x}{1+(x^2-x)^2}$ (b) $\frac{2x-1}{1+(x^2-x)^2}$

(c) $\frac{2x-1}{1-(x^2-x)^2}$ (d) $\frac{-2x+1}{1+(x^2-x)^2}$

(e) $(2x - 1)(1 + (x^2 - x)^2)$

**60.** The function $f(x) = x^2(x - 2)$ is strictly decreasing in

(a) (1, 2) (b) (–1, 1)

(c) $\left(\frac{4}{3}, \infty\right)$ (d) (–1, 0)

(e) $\left(0, \frac{4}{3}\right)$

**61.** The surface area of a solid hemisphere is increasing at the rate of 8 $cm^2$/sec (retaining its shape). Then the rate of change of its volume (in $cm^3$/sec), when the radius is 5 cm, is

(a) $\frac{50}{3}$ (b) $\frac{20}{3}$ (c) $\frac{40}{3}$ (d) $\frac{25}{3}$

(e) $\frac{80}{3}$

**62.** The function $f(x) = 2x^3 - 3x^2 - 36x + 28$ is increasing in

(a) $(-\infty, -1] \cup [3, \infty)$ (b) $(-\infty, -2] \cup [3, \infty)$

(c) $(-\infty, -2] \cup [5, \infty)$ (d) $(-\infty, -5] \cup [3, \infty)$

(e) $(-\infty, -2] \cup [8, \infty)$

**63.** Let $f(x) = x^2 + \alpha x + \beta$. If $f$ has a local minimum at (2, 6), then $f(0)$ is equal to

(a) 10 (b) –6 (c) 8 (d) –8

(e) 6

**64.** $\int \frac{2x^2+4x+3}{x^2+x+1}dx =$

(a) $2\log_e|x^2 + x + 1| + C$

(b) $2x\log_e|x^2 + x + 1| + C$

(c) $\frac{1}{2}\log_e|x^2 + x + 1| + C$

(d) $2x + \log_e|x^2 + x + 1| + C$

(e) $x + 2\log_e|x^2 + x + 1| + C$

**65.** $\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}dx =$

(a) $\frac{1}{2}(\sin^{-1}x)^2 + C$

(b) $-(\sin^{-1}x)\sqrt{1-x^2} + C$

(c) $(\sin^{-1}x)\sqrt{1-x^2} + x + C$

(d) $(\sin^{-1}x)\sqrt{1-x^2} - x + C$

(e) $(\sin^{-1}x)^2 + C$

**66.** $\int x^7(x^8+1)^{-3/4}dx =$

(a) $\frac{1}{2}\left(1+\frac{1}{x^8}\right)^{1/4} + C$ (b) $4\left(1+\frac{1}{x^8}\right)^{1/4} + C$

(c) $(x^8+1)^{1/4} + C$ (d) $4(x^8+1)^{1/4} + C$

(e) $\frac{1}{2}(x^8+1)^{1/4} + C$

**67.** $\int e^x \sec x(1+\tan x)dx =$

(a) $e^x\sec^2 x + C$ (b) $e^x\tan x + C$

(c) $e^x\sec x + C$ (d) $e^x\tan^2 x + C$

(e) $e^x\sec x\tan x + C$

**68.** $\int e^x\left(x^2-2\right)\cos\left(e^x\left(x^2-2x\right)\right)dx =$

(a) $\sin(e^x(x^2 - 2x)) + C$ (b) $\sin(e^x(x^2 - 2)) + C$

(c) $x^2e^x\sin(e^x(x^2 - 2)) + C$

(d) $e^x\sin(e^x(x^2 - 2)) + C$

(e) $e^x\sin(x^2e^x - 2xe^x) + C$

**69.** If $\int_{-\sqrt{3}}^{1}(-6x^2+18)dx = \alpha + \beta\sqrt{3}$ then the value of $\alpha + \beta$ is equal to

(a) 12 (b) 18 (c) 24 (d) 28

(e) 32

**70.** The value of $\int_{\pi/10}^{2\pi/5}\frac{\cot^3 x}{1+\cot^3 x}dx$ is equal to

(a) $\frac{\pi}{20}$ (b) $\frac{\pi}{10}$ (c) $\frac{3\pi}{20}$ (d) $\frac{\pi}{5}$

(e) $\frac{\pi}{4}$

**71.** The area of the region bounded by $y = x^{5/2}$ and $y = x$ (in square units) is

(a) $\frac{3}{7}$ (b) $\frac{2}{7}$ (c) $\frac{3}{14}$ (d) $\frac{5}{14}$

(e) $\frac{4}{7}$

**72.** $\int_0^1 \frac{3^{2x}}{3^{2x}+1}dx =$

(a) $\frac{\log_e 5}{2\log_e 3}$ (b) $\frac{\log_e 5}{9\log_e 3}$

(c) $\frac{\log_e 5}{3\log_e 3}$ (d) $\frac{2\log_e 5}{3\log_e 3}$

(e) $\frac{2\log_e 5}{9\log_e 3}$

**73.** If $y(x) = 2y'(x)$, $y(x) \geq 0$ and $y(0) = e^2$, then $y(x) =$

(a) $e^{x/2} + 2$ (b) $e^{2x}$ (c) $e^{x/2}$ (d) $e^2e^{x/2}$

(e) $e^{2x} + 2$

**74.** The integrating factor of the differential equation $\sin x\,dy = \frac{1}{2}(\sin 2x + 2y\cos x)dx$ is

(a) $\sec x$ (b) $\sin x$ (c) $\tan x$ (d) $\cos x$

(e) $\operatorname{cosec} x$

**75.** In the graphical method of a linear programming problem, the optimal solution lies

(a) at the centre of the feasible region

(b) at a corner point of the feasible region

(c) at a point on the $x$-axis

(d) at the origin

(e) at the point where the objective function is zero

## SOLUTIONS

**1. (e) :** Let number of elements in $A$, $B$ and $C$ are $m$, $n$ and $p$ respectively.

$\therefore$ $m \times n = 160$, $n \times p = 80$ and $p \times m = 200$

$\therefore$ On multiplying each expressions, we get

$(mnp)^2 = 160 \times 80 \times 200 \implies mnp = 1600$

$\implies m \times 80 = 1600 \implies m = 20$ $[\because np = 80]$

$\therefore$ $n(A) = 20$

**2. (b) :** Inverse image of 5 is $f(x) = 5$

$\implies x^2 - 10x - 19 = 5 \implies x^2 - 10x - 24 = 0$

$\implies x^2 - 12x + 2x - 24 = 0 \implies x(x - 12) + 2(x - 12) = 0$

$\implies (x - 12)(x + 2) = 0 \implies x = 12$ or $x = -2$

$\therefore$ $f^{-1}(5) = \{-2, 12\}$

**3. (e) :** We have, $f(x) = \cos x$

$$\therefore \frac{1}{2}\left[f(x+y)+f\left(y-x\right)\right]-f(x)f(y)$$

$$=\frac{1}{2}\left[\cos(x+y)+\cos\left(y-x\right)\right]-\cos x\cos y$$

$$=\frac{1}{2}\left[\cos x\cos y-\sin x\sin y+\cos y\cos x+\sin y\sin x\right]$$

$$-\cos x\cos y$$

$= \cos x\cos y - \cos x\cos y = 0$

**4. (d) :** We have, $f(x) = \log_5 x$ $(x > 0)$ and $g(x) = \cos^{-1} x$ $(-1 \le x \le 1)$

$\therefore$ $gof = g(f(x)) = \cos^{-1}(\log_5 x)$

As, $-1 \le \log_5 x \le 1$

$$\implies 5^{-1} \le x \le 5^1 \implies \frac{1}{5} \le x \le 5 \implies x \in \left[\frac{1}{5}, 5\right]$$

$\therefore$ Domain of $gof$ is $\left[\frac{1}{5}, 5\right]$

**5. (b) :** Given, $z = 1 + \frac{1}{i} \implies z = 1 - i$ $[\because i^2 = -1]$

$\therefore$ $z^2 = (1 - i)^2 = 1 + i^2 - 2i = 1 - 1 - 2i = -2i$

$\therefore$ $z^4 = (z^2)^2 = (-2i)^2 = 4i^2 = -4$ $[\because i^2 = -1]$

**6. (e) :** Let $z = \left(2\sqrt{2} + i\,2\sqrt{2}\right)^2 = \left(2\sqrt{2}\right)^2 (1+i)^2$

$\implies z = 8(1 + i^2 + 2i) \implies z = 8(1 - 1 + 2i)$ $[\because i^2 = -1]$

$\implies z = 8(2i) \implies z = 16i = 0 + 16i$

$\therefore$ $|z| = \sqrt{(0)^2 + (16)^2} = 16$

**7. (d) :** We have, $z + \overline{z} = 6$, and $z - \overline{z} = 4i$

$\therefore$ $z + \overline{z} + z - \overline{z} = 6 + 4i$

$\implies 2z = 6 + 4i \implies z = 3 + 2i$

$\therefore$ $|z| = \sqrt{(3)^2 + (2)^2} = \sqrt{9+4} = \sqrt{13} \implies |z|^2 = 13$

**8. (d) :** We have, $z = \frac{2 - i}{\alpha + i} \implies z = \frac{(2-i)(\alpha - i)}{(\alpha^2 + 1)}$

$$\implies z = \frac{(2\alpha - 1) - i(\alpha + 2)}{(\alpha^2 + 1)} = \frac{(2\alpha - 1)}{(\alpha^2 + 1)} - \frac{i(\alpha + 2)}{(\alpha^2 + 1)}$$

$$\therefore \overline{z} = \frac{(2\alpha - 1)}{(\alpha^2 + 1)} + \frac{i(\alpha + 2)}{(\alpha^2 + 1)}$$

Also, $4\,\mathrm{Re}(z) = 3\,\mathrm{Im}(\overline{z})$

$$\implies 4\left(\frac{2\alpha - 1}{\alpha^2 + 1}\right) = 3\left(\frac{\alpha + 2}{\alpha^2 + 1}\right) \implies 4(2\alpha - 1) = 3(\alpha + 2)$$

$\implies 8\alpha - 4 = 3\alpha + 6 \implies 5\alpha = 10 \implies \alpha = 2$

**9. (e) :** Let the first term and common ratio of the G.P. be '$a$' and '$r$' respectively.

$\therefore$ $a = 4$, $a_3 = ar^2 = 8$

$\implies 4 \times r^2 = 8 \implies r^2 = 2 \implies r = \pm\sqrt{2}$

$[\because$ G.P. is increasing$]$

$\implies r = \sqrt{2}$

$$\therefore a_{21} = ar^{20} = 4 \times \left(\sqrt{2}\right)^{20} = 4 \times 2^{10} = 4 \times 1024 = 4096$$

**10. (d) :** Let the common ratio be '$r$'.

Given, $a_1 a_2 a_3 = 64$

$\implies a_1 \times a_1 r \times a_1 r^2 = 64 \implies a_1^3 r^3 = 64 \implies a_1 r = 4$ ...(i)

Now, $a_1 a_2 a_3 a_4 a_5 = 32$ (Given)

$\implies a_1 \times a_1 r \times a_1 r^2 \times a_1 r^3 \times a_1 r^4 = 32$

$\implies (a_1 r^2)^5 = 32 \implies a_1 r^2 = 2$ ...(ii)

From (i) and (ii), we get $r = \frac{1}{2}$

**11. (c) :** Given, $t_n = \frac{n(n+6)}{(n+4)}$

Also, $t_n = 5 \implies \frac{n(n+6)}{(n+4)} = 5$

$\implies n^2 + 6n = 5n + 20 \implies n^2 + n - 20 = 0$

$\implies n^2 + 5n - 4n - 20 = 0 \implies n(n + 5) - 4(n + 5) = 0$

$\implies (n + 5)(n - 4) = 0 \implies n = 4, -5$

$\therefore$ $n = 4$ $[\because$ '$n$' can't be negative$]$

**12. (a) :** Let the first term be '$a$' and common ratio be '$r$'.

$\therefore$ $a \times ar \times ar^2 \times ar^3 \times ar^4 = 32$ $[\because r > 0]$

$\implies a^5 \times r^{10} = 32 \implies ar^2 = 2$

$\therefore$ $a_3 = 2$

**13. (e)**

**14. (b) :** Given, $\alpha = {}^nC_r$ and $\beta = {}^nC_{r-1}$

$$\therefore 1+\frac{\alpha}{\beta}=\frac{\alpha+\beta}{\beta}=\frac{{}^nC_r+{}^nC_{r-1}}{{}^nC_{r-1}}$$

$$=\frac{{}^{n+1}C_r}{{}^nC_{r-1}}=\frac{\dfrac{(n+1)!}{(n+1-r)!\times r!}}{\dfrac{n!}{(n-r+1)!\times(r-1)!}}=\frac{n+1}{r}$$

**15. (d):** We have, ${}^{11}P_r = 7920 \Rightarrow \dfrac{11!}{(11-r)!} = 7920$

$\Rightarrow \dfrac{11\times10\times9\times8\times7!}{(11-r)!} = 11\times10\times9\times8$

$\Rightarrow 7! = (11-r)! \Rightarrow r = 4$

**16. (c):** General term in the expansion of $(x + a)^n$ is

$T_{r+1} = {}^nC_r x^{n-r}a^r$

$T_{6+1} = {}^8C_6 \times 4x^2\alpha^6$ ...(i)

$T_{5+1} = {}^8C_5 \times 8x^3\alpha^5$ ...(ii)

Since, coefficients of $x^2$ and $x^3$ are equal.

$\Rightarrow 4 \times {}^8C_6\,\alpha^6 = 8 \times {}^8C_5\,\alpha^5 \Rightarrow \dfrac{8!\times\alpha}{6!\times2!} = \dfrac{8!\times2}{3!\times5!}$

$\Rightarrow \alpha = 4$

**17. (b):** We have, $A = \{0, 2, 4, 6, 8\}$

Number of 5-digit numbers formed without replacement

$= 4 \times 4 \times 3 \times 2 \times 1 = 96$

**18. (d):** $A$ is a $3 \times 3$ matrix

$\therefore$ $B$ is also a $3 \times 3$ matrix. [$\because$ $B = 3A$]

$\therefore$ $|B| = |3A| \Rightarrow |B| = 3^3|A|$

$\Rightarrow |B| = 27 \times 5 \Rightarrow |B| = 135$

$\therefore$ $|\text{adj } B| = |B|^{3-1} = |B|^2 = (135)^2$

Also, $|3A| = |B| = 135$ $\therefore$ $\dfrac{|\text{adj } B|}{|3A|} = \dfrac{(135)^2}{135} = 135$

**19. (e):** We have, $\begin{pmatrix}-1 & 2\\ 3 & -4\\ -5 & 6\end{pmatrix}\begin{pmatrix}7\\ 8\end{pmatrix} = \begin{pmatrix}\alpha\\ \beta\\ 13\end{pmatrix}$

$\Rightarrow \begin{pmatrix}-7+16\\ 21-32\\ -35+48\end{pmatrix} = \begin{pmatrix}\alpha\\ \beta\\ 13\end{pmatrix} \Rightarrow \begin{pmatrix}9\\ -11\\ 13\end{pmatrix} = \begin{pmatrix}\alpha\\ \beta\\ 13\end{pmatrix}$

$\Rightarrow \alpha = 9, \beta = -11$ $\therefore$ $\alpha + \beta = 9 + (-11) = -2$

**20. (a):** Let $A = \begin{vmatrix}8-k & 2\\ -2 & 4-k\end{vmatrix}$

Since, $A$ is a singular matrix

So, $|A| = 0$

$\Rightarrow (8 - k)(4 - k) + 4 = 0 \Rightarrow k^2 - 12k + 36 = 0$

$\Rightarrow (k - 6)^2 = 0 \Rightarrow k = 6$

**21. (d):** If a system of linear equations has no unique solution, then the determinant of coefficient matrix is zero.

$\therefore$ $\begin{vmatrix}1 & 1 & 1\\ 2 & 3 & -m\\ 3 & 5 & 3\end{vmatrix} = 0$

$\Rightarrow 1(9 + 5m) - 1(6 + 3m) + 1(10 - 9) = 0$

$\Rightarrow 9 + 5m - 6 - 3m + 1 = 0 \Rightarrow 2m + 4 = 0$

$\Rightarrow 2m = -4 \Rightarrow m = -2$

**22. (d):** We have, $-4 \le 2 - 3x < 7$

$\Rightarrow -7 < 3x - 2 \le 4 \Rightarrow -5 < 3x \le 6$

$\Rightarrow -\dfrac{5}{3} < x \le 2$ $\therefore$ $x \in \left(-\dfrac{5}{3}, 2\right]$

**23. (e):** Given $-5 < x \le -1$

$\Rightarrow -21 < 5x + 4 \le b$ ...(i)

Now, $-5 < x \le -1$

$\Rightarrow -25 < 5x \le -5 \Rightarrow -25 + 4 < 5x + 4 \le -5 + 4$

$\Rightarrow -21 < 5x + 4 \le -1$ ...(ii)

$\therefore$ $b = -1$ [Comparing (i) and (ii)]

**24. (d):** $\tan15° + \tan75° = \tan15° + \tan(90° - 15°)$

$= \tan15° + \cot15°$

$= \dfrac{\sin15°}{\cos15°} + \dfrac{\cos15°}{\sin15°} = \dfrac{\sin^2 15° + \cos^2 15°}{\sin15°\cos15°}$

$= \dfrac{1}{\sin15°\cos15°} \times \dfrac{2}{2} = \dfrac{2}{\sin30°} = 2\times2 = 4$

**25. (a):** Given, $x + z = 2y$

$\Rightarrow x + z = 2 \times \dfrac{\pi}{4} \Rightarrow x + z = \dfrac{\pi}{2} \Rightarrow x = \left(\dfrac{\pi}{2} - z\right)$

$\Rightarrow \tan x = \tan\left(\dfrac{\pi}{2} - z\right) \Rightarrow \tan x = \cot z$

$\Rightarrow \tan x \tan z = 1$ ...(i)

$\therefore$ $\tan x \tan y \tan z = (\tan x \tan z)\tan\dfrac{\pi}{4} = 1$ [From (i)]

**26. (c):** We have, $\sin x + \sin y = a$

$\Rightarrow 2\sin\left(\dfrac{x+y}{2}\right)\cos\left(\dfrac{x-y}{2}\right) = a$ ...(i)

and $\cos x + \cos y = b$

$\Rightarrow 2\cos\left(\dfrac{x+y}{2}\right)\cos\left(\dfrac{x-y}{2}\right) = b$ ...(ii)

From (i) and (ii), we get

$\tan\left(\dfrac{x+y}{2}\right) = \dfrac{a}{b} \Rightarrow \tan\left(\dfrac{\pi}{3}\right) = \dfrac{a}{b} \Rightarrow \dfrac{a}{b} = \sqrt{3}$

**27. (c):** Given, $\sin\alpha = \dfrac{12}{13}$

$\therefore$ $\cos^2\alpha = 1 - \sin^2\alpha = 1 - \left(\dfrac{12}{13}\right)^2 = \dfrac{169-144}{169} = \dfrac{25}{169}$

$\Rightarrow \cos\alpha = \pm\dfrac{5}{13} \Rightarrow \cos\alpha = -\dfrac{5}{13}$ $\left[\because \dfrac{\pi}{2} < \alpha < \dfrac{3\pi}{2}\right]$

Now, $\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha} = -\dfrac{12}{5}$

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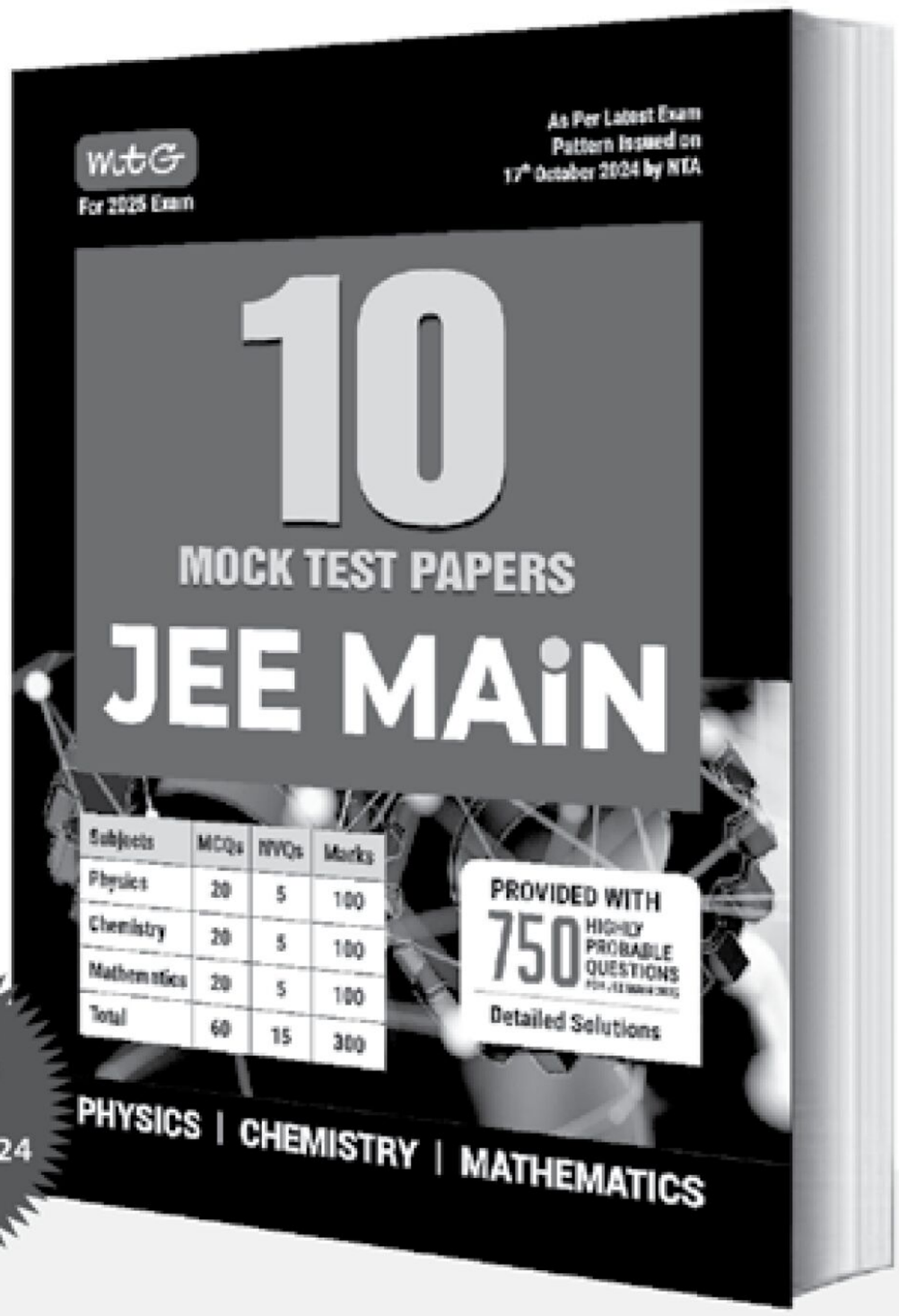


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**28. (c) :** We have, $f(x)=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$

$$\therefore \quad f\left(\frac{1}{\sqrt{3}}\right)=\tan^{-1}\left(\frac{2\times\frac{1}{\sqrt{3}}}{1-\frac{1}{3}}\right)=\tan^{-1}\left(\frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}}\right)$$

$$=\tan^{-1}\left(\sqrt{3}\right) \ =\tan^{-1}\left(\tan\frac{\pi}{3}\right)=\frac{\pi}{3}$$

**29. (c) :** We have, $5\sin^{-1}\alpha+3\cos^{-1}\alpha=\pi$

$\Rightarrow \quad 2\sin^{-1}\alpha+3(\sin^{-1}\alpha+\cos^{-1}\alpha)=\pi$

$$\Rightarrow \quad 2\sin^{-1}\alpha+\frac{3\pi}{2}=\pi \Rightarrow 2\sin^{-1}\alpha=-\frac{\pi}{2}$$

$$\Rightarrow \quad \sin^{-1}\alpha=-\frac{\pi}{4}$$

$$\Rightarrow \quad \alpha=\sin\left(-\frac{\pi}{4}\right)=-\sin\frac{\pi}{4}=-\frac{1}{\sqrt{2}}$$

**30. (c) :** Given, $\theta=\cot^{-1}\sqrt{\frac{1-x}{1+x}}$

$$\Rightarrow \quad \cot\theta=\sqrt{\frac{1-x}{1+x}} \quad \Rightarrow \quad \tan\theta=\sqrt{\frac{1+x}{1-x}}$$

Since, $\sec^2\theta=1+\tan^2\theta=1+\left(\frac{1+x}{1-x}\right)=\frac{(1-x)+(1+x)}{(1-x)}$

$$\Rightarrow \quad \sec^2\theta=\frac{2}{(1-x)}$$

**31. (b) :** Since, $ax+by+c=0$ passes through the point $(-10, 7)$.

$\therefore \quad -10a+7b+c=0$ ...(i)

A line perpendicular to the line $ax+by+c=0$ is $bx-ay=\lambda$

On comparing with $11x-7y=13$, we get

$a=7$ and $b=11$.

$\therefore \quad -70+77+c=0 \Rightarrow c=-7$ [From (i)]

**32. (e) :** Let $AD$ be the altitude from vertex $A$ on $BC$.

$$\therefore \quad AD=\left|\frac{(-2)+(2)-6}{\sqrt{(1)^2+(1)^2}}\right|=\left|\frac{-6}{\sqrt{2}}\right| \quad \Rightarrow \quad AD=\frac{6}{\sqrt{2}}=3\sqrt{2}$$

$$\therefore \text{ Side of triangle}=\frac{2}{\sqrt{3}}\times\text{Altitude}=\frac{2}{\sqrt{3}}\times3\sqrt{2}=2\sqrt{6}$$

**33. (d) :** Given, $x^2-4x+8y+4=0$

$\Rightarrow \quad 8y+(x-2)^2=0 \Rightarrow \quad (x-2)^2=-8y$

Comparing it with $(x-h)^2=-4a(y-k)$, we get

$h=2, a=2, k=0$

$\therefore$ Focus of the parabola $=(h, k-a)=(2, -2)$

**34. (a) :** Let '$O$' be the centre of the circle.

Equation of $x$-axis is $y=0$.

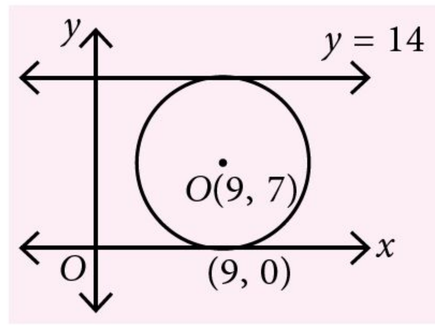
$\Rightarrow$ circle touches the parallel lines $y=0$ and $y=14$.

$\therefore \quad O(9, 7)$ and radius $=7$

$\therefore$ Equation of circle is

$(x-9)^2+(y-7)^2=(7)^2$

$\Rightarrow \quad (x-9)^2+(y-7)^2=49$




**35. (e) :** Given, $m^2-n^2=45$ ...(i)

$\therefore \quad m>n$

$\Rightarrow \quad 2a=m, 2b=n$

From (i), $4a^2-4b^2=45 \Rightarrow (a^2-b^2)=\frac{45}{4}$ ...(ii)

Also, $e^2=\frac{a^2-b^2}{a^2} \Rightarrow \frac{5}{9}=\frac{45}{4a^2}$ (Using (ii))

$$\Rightarrow \quad 4a^2=81 \Rightarrow a^2=\frac{81}{4} \quad \therefore \quad a=\frac{9}{2} \Rightarrow 2a=9$$

$\therefore$ Length of major axis $=9$

**36. (a) :** Given, $4y=x^2-6x+17$

$\Rightarrow \quad 4y=x^2-6x+9+8 \Rightarrow 4y-8=(x-3)^2$

$\Rightarrow \quad (x-3)^2=4(y-2)$ ...(i)

Comparing (i) with $(x-h)^2=4a(y-k)$, we get

$h=3, k=2, a=1$

$\therefore$ Vertex of the parabola, $(h, k)=(3, 2)$

**37. (c)**

**38. (e) :** Given, $\vec{a}+\vec{b}$ is perpendicular to $\vec{a}-\vec{b}$.

$$\therefore \quad (\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b})=0$$

$$\Rightarrow \quad (\lambda\hat{i}+16\hat{j}-18\hat{k})\cdot(2\hat{i}+8\hat{j}+\lambda\hat{k})=0$$

$\Rightarrow \quad 2\lambda+128-18\lambda=0 \quad \Rightarrow \quad \lambda=8$

Now, $2\vec{a}=(\vec{a}+\vec{b})+(\vec{a}-\vec{b})=(\lambda+2)\hat{i}+24\hat{j}+(\lambda-18)\hat{k}$

$$\Rightarrow \quad 2\vec{a}=10\hat{i}+24\hat{j}-10\hat{k} \quad \Rightarrow \quad \vec{a}=5\hat{i}+12\hat{j}-5\hat{k}$$

$$\Rightarrow \quad |\vec{a}|=\sqrt{(5)^2+(12)^2+(-5)^2}=\sqrt{194}$$

**39. (b) :** Given, $|\vec{a}|=12$ and projection of $\vec{a}$ on $\vec{b}=6\sqrt{3}$

$$\Rightarrow \quad \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}=6\sqrt{3} \quad \Rightarrow \quad \frac{|\vec{a}||\vec{b}|\cos\theta}{|\vec{b}|}=6\sqrt{3}$$

$$\Rightarrow \quad \cos\theta=\frac{6\sqrt{3}}{|\vec{a}|}=\frac{6\sqrt{3}}{12}=\frac{\sqrt{3}}{2}=\cos\frac{\pi}{6} \quad \Rightarrow \quad \theta=\frac{\pi}{6}$$

**40. (e) :** Since, $\vec{b}$ is parallel to $\vec{a}$.

$$\therefore \ \vec{b}=\lambda\vec{a} \quad \Rightarrow \quad \vec{b}=\lambda(6\hat{i}+2\hat{j}+3\hat{k})=6\lambda\hat{i}+2\lambda\hat{j}+3\lambda\hat{k}$$

Also, $\vec{a}\cdot\vec{b}=\frac{49}{2} \quad \Rightarrow 36\lambda+4\lambda+9\lambda=\frac{49}{2} \Rightarrow \lambda=\frac{1}{2}$

$\therefore\ \vec{b} = 3\hat{i} + \hat{j} + \frac{3}{2}\hat{k}$

$\Rightarrow\ |\vec{b}| = \sqrt{(3)^2 + (1)^2 + \left(\frac{3}{2}\right)^2} = \sqrt{9 + 1 + \frac{9}{4}} = \sqrt{\frac{49}{4}} = \frac{7}{2}$

**41. (a) :** Given, $\left|\vec{a} + \vec{b}\right| = \frac{\sqrt{14}}{2}$

$\therefore\ \left|\vec{a} + \vec{b}\right|^2 = \frac{14}{4} \Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b} = \frac{14}{4}$

$\Rightarrow\ 1 + 1 + 2\vec{a}\cdot\vec{b} = \frac{7}{2} \Rightarrow 2\vec{a}\cdot\vec{b} = \frac{7}{2} - 2 = \frac{3}{2} \Rightarrow 4\vec{a}\cdot\vec{b} = 3$

Now, $\left|\vec{a} + \vec{b}\right|^2 - \left|\vec{a} - \vec{b}\right|^2 = 4\vec{a}\cdot\vec{b} = 3$

**42. (b)**

**43. (c) :** Let the point $P(6,1,3)$ meets the line

$l_1 : \frac{x-1}{2} = \frac{y}{1} = \frac{z-2}{3} = \lambda$ at $Q$

$\therefore\ x = 2\lambda + 1, y = \lambda, z = 3\lambda + 2$

Since, $Q$ be on the line $l_1$.

$\therefore\ Q \equiv (2\lambda + 1, \lambda, 3\lambda + 2)$

$\therefore$ Direction vector of line $PQ = Q - P$

$= (2\lambda - 5)\hat{i} + (\lambda - 1)\hat{j} + (3\lambda - 1)\hat{k}$

Also, direction ratios of line $l_1$ is $< 2, 1, 3 >$

Since, $PQ \perp l_1$

$\Rightarrow\ 4\lambda - 10 + \lambda - 1 + 9\lambda - 3 = 0$

$\Rightarrow\ 14\lambda - 14 = 0 \Rightarrow\ 14\lambda = 14 \Rightarrow\ \lambda = 1$

$\therefore$ Coordinates of $Q$ are (3, 1, 5).

**44. (d):** Given, $a_1 = 1, b_1 = -1, c_1 = -1, a_2 = 2, b_2 = 2$ and $c_2 = -2$.

$\therefore$ Angle between the lines is given by

$$\cos\theta = \left|\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}\right|$$

$$\Rightarrow\ \cos\theta = \left|\frac{2 - 2 + 2}{\sqrt{1^2 + 1^2 + 1^2}\sqrt{2^2 + 2^2 + 2^2}}\right|$$

$$\Rightarrow\ \cos\theta = \left|\frac{2}{\sqrt{3} \times 2\sqrt{3}}\right| = \left|\frac{1}{3}\right| \Rightarrow\ \theta = \cos^{-1}\left(\frac{1}{3}\right)$$

**45. (e) :** Let the vector be $\vec{a}$ which is parallel to the line passing through the points $P(10, 8, 6)$ and $Q(13, 9, 4)$.

$\therefore$ Dr's of $\vec{a} = (3, 1, -2)\ \therefore\ \vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$

and $|\vec{a}| = \sqrt{(3)^2 + (1)^2 + (-2)^2} = \sqrt{14}$

$\therefore\ \hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{3\hat{i} + \hat{j} - 2\hat{k}}{\sqrt{14}}$

**46. (a)**

**47. (c) :** We have, $P(B) = 0.7$

$\therefore\ P(B') = 1 - P(B) = 1 - 0.7 = 0.3$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = 0.4$$

$\Rightarrow\ P(A \cap B) = 0.4 \times 0.7 = 0.28$ ...(i)

$$P(A \mid B') = \frac{P(A \cap B')}{P(B')} = 0.7$$

$\Rightarrow\ P(A \cap B') = 0.7 \times 0.3 = 0.21$ ...(ii)

$\therefore\ P(A) = P(A \cap B) + P(A \cap B') = 0.28 + 0.21 = 0.49$

**48. (d) :** since, $= \left(\frac{1}{n}\sqrt{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2}\right)$

$$= \frac{1}{4}\left[\sqrt{4\left((-3)^2 + 0^2 + 3^2 + 8^2\right) - (-3 + 0 + 3 + 8)^2}\right]$$

$$= \frac{1}{4}\left[\sqrt{4(82) - 64}\right] = \frac{1}{4}\sqrt{264} = \frac{\sqrt{66}}{2}$$

**49. (c) :** The probability of getting '5', when an unbiased die is tossed, $p = \frac{1}{6}$

$\therefore\ q = 1 - p = 1 - \frac{1}{6} = \frac{5}{6}$

Since, $X$ follows the geometric distribution.

$\therefore\ P(X = n) = q^{n-1}p$

$\therefore\ P(X = 2) = \left(\frac{5}{6}\right)^1 \times \left(\frac{1}{6}\right) = \frac{5}{36}$

and $P(X = 5) = \left(\frac{5}{6}\right)^4 \times \left(\frac{1}{6}\right) = \frac{625}{(6)^5}$

$\therefore\ \frac{P(X = 2)}{P(X = 5)} = \frac{\frac{5}{36}}{\frac{625}{6^5}} = \frac{216}{125}$

**50. (a)**

**51. (c) :** Since, the given function is continuous at $x = 0$

$\therefore\ \lim_{x \to 0} f(x) = 5$

$$\Rightarrow\ \lim_{x \to 0} \frac{\tan\alpha x + (\beta + 1)\tan x}{x} = 5$$

$$\Rightarrow\ \lim_{x \to 0} \alpha\left(\frac{\tan\alpha x}{\alpha x}\right) + \lim_{x \to 0}(\beta + 1)\frac{\tan x}{x} = 5$$

$$\Rightarrow \quad \alpha \lim_{x\to 0}\left(\frac{\tan \alpha x}{\alpha x}\right)+(\beta+1)\lim_{x\to 0}\frac{\tan x}{x}=5$$

$$\Rightarrow \quad \alpha+(\beta+1)=5 \Rightarrow \alpha+\beta=4$$

**52. (c) :** Given, $f(x)=\sqrt{x-3}+4\sqrt{5-x}$

$f(x)$ is defined when $x-3\geq 0$ and $5-x\geq 0$

$\Rightarrow \quad x\geq 3$ and $x\leq 5 \quad \therefore \quad D_f\in[3,5]$

**53. (d):** We have, $f(x)=\dfrac{3^x}{3^x+\sqrt{3}}$

$$\therefore \quad f(1-x)=\frac{3^{1-x}}{3^{1-x}+\sqrt{3}}=\frac{\frac{3}{3^x}}{\frac{3+\sqrt{3}.3^x}{3^x}}$$

$$\Rightarrow \quad f(1-x)=\frac{3}{3+\sqrt{3}.3^x}=\frac{\sqrt{3}}{\sqrt{3}+3^x}$$

$$\therefore \quad f(x)+f(1-x)=\frac{3^x}{3^x+\sqrt{3}}+\frac{\sqrt{3}}{\sqrt{3}+3^x}=\frac{3^x+\sqrt{3}}{\sqrt{3}+3^x}=1$$

**54. (b):** We have,

$$\lim_{x\to 0}\frac{\sqrt{\cos^2 x+3}-\sqrt{\cos^2 x+\sin x+3}}{x}$$

$$=\lim_{x\to 0}\frac{\cos^2 x+3-\cos^2 x-\sin x-3}{x\left(\sqrt{\cos^2 x+3}+\sqrt{\cos^2 x+\sin x+3}\right)}$$

$$=\lim_{x\to 0}\frac{-\sin x}{x\left(\sqrt{\cos^2 x+3}+\sqrt{\cos^2 x+\sin x+3}\right)}$$

$$=\frac{-1}{\left(\sqrt{1+3}+\sqrt{1+0+3}\right)}=\frac{-1}{2+2}=-\frac{1}{4}$$

**55. (d):** $f(x)=|x^2+x-6|=|(x+3)(x-2)|$

So, the given function is not differentiable at $x=-3$ and $x=2$.

$\therefore \quad a=-3$ and $b=2$.

$\therefore \quad a^2+b^2=(-3)^2+(2)^2=9+4=13$

**56. (a):** Given, $\dfrac{\pi}{6}\leq x\leq\dfrac{\pi}{3} \Rightarrow \dfrac{\pi}{2}\leq 3x\leq\pi$

$\sin 3x\geq 0$ and $\cos 3x\leq 0$

$\therefore \quad f(x)=|\sin 3x|-|\cos 3x|$, we write as

$f(x)=\sin 3x+\cos 3x$

Now, $f'(x)=\cos 3x\times 3-\sin 3x\times 3$

$\therefore \quad f'(x)=3[\cos 3x-\sin 3x]$

$$f'\left(\frac{\pi}{4}\right)=3\left[\cos\frac{3\pi}{4}-\sin\frac{3\pi}{4}\right]$$

$$=3\times\left[-\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}\right]=3\times(-\sqrt{2})=-3\sqrt{2}$$

**57. (b):** We have, $h(x)=f\left(\sqrt{g(x)}\right)$

$$\Rightarrow \quad h'(x)=\frac{f'\left(\sqrt{g(x)}\right)g'(x)}{2\sqrt{g(x)}}$$

$$\therefore \quad h'(3)=\frac{f'\left(\sqrt{g(3)}\right)g'(3)}{2\sqrt{g(3)}}=\frac{f'(3)\times 3}{2\times 3}=\frac{6\times 3}{6}=3$$

**58. (c)**

**59. (b):** We have, $y=\tan^{-1}(x^2-x)$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{1}{1+(x^2-x)^2}\times\frac{d}{dx}\left(x^2-x\right)=\frac{(2x-1)}{1+(x^2-x)^2}$$

**60. (e) :** We have, $f(x)=x^2(x-2)=x^3-2x^2$

$\Rightarrow \quad f'(x)=3x^2-4x$

Since, $f'(x)<0$ [$\because$ $f(x)$ is strictly decreasing function]

$$\Rightarrow \quad 3x^2-4x<0 \Rightarrow x(3x-4)<0 \Rightarrow x\in\left(0,\frac{4}{3}\right)$$

**61. (c) :** Let '$r$' be the radius of the solid hemisphere.

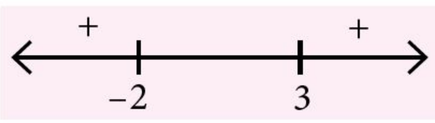
Since, $\dfrac{dA}{dt}=8 \Rightarrow \dfrac{d}{dt}\left(3\pi r^2\right)=8 \Rightarrow 3\pi\times 2r\times\dfrac{dr}{dt}=8$

$$\Rightarrow \quad \left.\frac{dr}{dt}\right|_{r=5}=\frac{8}{6\pi\times 5}=\frac{4}{15\pi}$$

Now, $V=\dfrac{2}{3}\pi r^3$

$$\therefore \quad \frac{dV}{dt}=\frac{2}{3}\pi\left(3r^2\right)\frac{dr}{dt}=2\pi(5)^2\times\frac{4}{15\pi}=\frac{40}{3}\ \text{cm}^2/\text{sec}$$

**62. (b):** Given $f(x)=2x^3-3x^2-36x+28$ is increasing function.




$\therefore \quad f'(x)\geq 0$

$\Rightarrow \quad 6x^2-6x-36\geq 0$

$\Rightarrow \quad x^2-x-6\geq 0$

$\Rightarrow \quad x^2-3x+2x-6\geq 0 \Rightarrow x(x-3)+2(x-3)\geq 0$

$\Rightarrow \quad (x-3)(x+2)\geq 0 \Rightarrow x\in(-\infty,-2]\cup[3,\infty)$

**63. (a) :** We have, $f(x)=x^2+\alpha x+\beta$

Since, it is an opening upward parabola as coefficient of $x^2$ is positive. So, it attains local minimum at its vertex.

$$\therefore \quad \frac{-b}{2a}=2 \quad \Rightarrow \quad \frac{-\alpha}{2}=2 \Rightarrow \alpha=-4$$

$\because \quad f(2)=6$

$\Rightarrow \quad 4+2(-4)+\beta=6 \Rightarrow 4-8+\beta=6 \Rightarrow \beta=10$

$\therefore \quad f(0)=(0)^2+(-4)\times 0+10 \Rightarrow f(0)=10$

**64. (d):** Let $I=\displaystyle\int\frac{2x^2+4x+3}{x^2+x+1}dx$

$$=\int\frac{\left(2x^2+2x+2\right)}{\left(x^2+x+1\right)}dx+\int\frac{2x+1}{\left(x^2+x+1\right)}dx$$

Put $x^2 + x + 1 = t \Rightarrow (2x + 1)dx = dt$

$\therefore \quad I = \int 2 \cdot dx + \int \frac{dt}{t} = 2x + \log_e |t| + C$

$= 2x + \log_e|x^2 + x + 1| + C$

**65. (a) :** Let $I = \int \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx$

Put $\sin^{-1}x = t \Rightarrow \frac{1}{\sqrt{1-x^2}} dx = dt$

$\therefore \quad I = \int t \cdot dt = \frac{t^2}{2} + C = \frac{(\sin^{-1} x)^2}{2} + C$

**66. (e) :** Let $I = \int x^7 (x^8 + 1)^{-3/4} dx$

Put $x^8 + 1 = t \Rightarrow 8x^7 dx = dt$

$\therefore \quad I = \frac{1}{8}\int t^{-3/4} dt = \frac{1}{8} \times \frac{(t)^{1/4}}{1/4}$

$= \frac{4}{8}(x^8 + 1)^{1/4} + C = \frac{1}{2}(x^8 + 1)^{1/4} + C$

**67. (c) :** We have, $\int e^x \sec x(1 + \tan x)dx$

$= \int e^x [\sec x + \sec x \tan x)] dx$

$= e^x \sec x + C \quad \left[\because \int e^x \left(f(x) + f'(x)\right)dx = e^x f(x) + c\right]$

**68. (a) :** Let $I = \int e^x \left(x^2 - 2\right)\cos\left(e^x \left(x^2 - 2x\right)\right)dx$

Put $e^x(x^2 - 2x) = t$

$\Rightarrow \quad (e^x(2x - 2) + (x^2 - 2x)e^x)\, dx = dt$

$\Rightarrow \quad [e^x(2x - 2 + x^2 - 2x)]dx = dt \Rightarrow e^x(x^2 - 2)\, dx = dt$

$\therefore \quad I = \int \cos t\, dt = \sin t + C = \sin(e^x (x^2 - 2x)) + C$

**69. (d):** Let $I = \int_{-\sqrt{3}}^{1} (-6x^2 + 18)dx$

$= \left[\frac{-6x^3}{3}\right]_{-\sqrt{3}}^{1} + 18[x]_{-\sqrt{3}}^{1}$

$= -2\left[1 + 3\sqrt{3}\right] + 18\left[1 + \sqrt{3}\right]$

$\Rightarrow \quad I = 16 + 12\sqrt{3} = \alpha + \beta\sqrt{3}$ (Given)

$\Rightarrow \quad \alpha = 16$ and $\beta = 12$

$\therefore \quad \alpha + \beta = 16 + 12 = 28$ (On comparing)

**70. (c)**

**71. (c) :** We have, $y = x^{5/2}$ ...(i) and $y = x$ ...(ii)

From (i) and (ii), we get $cx^{5/2} = x$ ; $x^{5/2} - x = 0$

$\Rightarrow \quad x(x^{3/2} - 1) = 0 \Rightarrow x = 0$ or $x^{3/2} = 1 \Rightarrow x = 1$

$\therefore \quad$ The intersection points are at $x = 0$ and $x = 1$

But $x \geq x^{5/2}$ for $0 \leq x \leq 1$.

$\therefore \quad$ Required area $= \int_0^1 (x - x^{5/2})dx$

$= \left[\frac{x^2}{2} - \frac{2x^{7/2}}{7}\right]_0^1 = \frac{1}{2} - \frac{2}{7} = \frac{7-4}{14} = \frac{3}{14}$ sq.units

**72. (a) :** Let $I = \int_0^1 \frac{3^{2x}}{3^{2x} + 1} dx$

Put $3^{2x} + 1 = u \quad \Rightarrow \quad 3^{2x} \times 2 \times \log_e 3\, dx = du$

$\Rightarrow \quad 3^{2x} dx = \frac{1}{2\log_e 3} du$

When $x = 0$, $u = 2$

When $x = 1$, $u = 10$

$\therefore \quad I = \int_2^{10} \frac{du}{2u \log_e 3} = \frac{1}{2\log_e 3} \int_2^{10} \frac{du}{u}$

$\Rightarrow \quad I = \frac{1}{2\log_e 3}\left(\log_e 10 - \log_e 2\right) = \frac{\log_e 5}{2\log_e 3}$

**73. (d):** We have, $y(x) = 2y'(x)$

$\Rightarrow \quad y(x) = 2\frac{dy}{dx} \Rightarrow \frac{dy}{y} = \frac{dx}{2}$

Integrating both sides, we get

$\int \frac{dy}{y} = \int \frac{dx}{2} \Rightarrow \log y = \frac{1}{2}x + c \Rightarrow y = e^{\frac{1}{2}x + c}$ ...(i)

$\therefore \quad e^2 = e^c \quad [\because y(0) = e^2]$

$\Rightarrow \quad c = 2$

Putting value of '$c$' in (i), we get

$y = e^{\frac{1}{2}x + 2} \Rightarrow y = e^{x/2}e^2$

**74. (e) :** Given, $\sin x\, dy = \frac{1}{2}(\sin 2x + 2y \cos x)dx$

$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{2}\left(\frac{\sin 2x + 2y\cos x}{\sin x}\right) = \cos x + y \cot x$

$\Rightarrow \quad \frac{dy}{dx} - y\cot x = \cos x$

$\therefore \quad$ I.F. $= e^{\int P(x)dx} = e^{\int -\cot x\, dx} = e^{\log \operatorname{cosec} x} = \operatorname{cosec} x$

**75. (b):** In the graphical method of a linear programming problem, the optimal solution always lies at a corner points of the feasible region.

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**MONTHLY TEST DRIVE CLASS XI ANSWER KEY**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.** | (a) | **2.** | (d) | **3.** | (c) | **4.** | (a) | **5.** | (a) |
| **6.** | (c) | **7.** | (a,b) | **8.** | (a,c) | **9.** | (b,d) | **10.** | (b,d) |
| **11.** | (b,c) | **12.** | (a,c,d) | **13.** | (a,d) | **14.** | (d) | **15.** | (d) |
| **16.** | (b) | **17.** | (1) | **18.** | (3) | **19.** | (14) | **20.** | (50) |

# GEAR UP for JEE 2026

## MULTIPLE CHOICE QUESTIONS

**1.** If $f(x)=\begin{cases}-x+1 & , x\le 0\\ -(x-1)^2, & x\ge 1\end{cases}$, then the number of solutions of $f(x)-f^{-1}(x)=0$ is

(a) 0 (b) 2
(c) 3 (d) 4

**2.** If $n\in N$, then $121^n-25^n+1900^n-(-4)^n$ is divisible by

(a) 1904 (b) 2000
(c) 2002 (d) 2006

**3.** If $f(x)=ax^2+bx+c,\ a,\ b,\ c\in R$ and the equation $f(x)-x=0$ has imaginary roots $\alpha$ and $\beta$, $\gamma$ and $\delta$ be the roots of $f(f(x))-x=0$, then $\begin{vmatrix}2 & \alpha & \delta\\ \beta & 0 & \alpha\\ \gamma & \beta & 1\end{vmatrix}$ is

(a) 0 (b) purely real
(c) purely imaginary (d) none of these

**4.** The area common to the curves $x^2+y^2=2$ and $x^2+4y^2=4$ is

(a) $\tan^{-1}2\sqrt{2}$ (b) $2\tan^{-1}2\sqrt{2}$
(c) $4\tan^{-1}\sqrt{2}$ (d) $4\tan^{-1}2\sqrt{2}$

**5.** The number of integral solutions of the equation $2x+y+z=10,\ x\ge 0,\ y\ge 0,\ z\ge 0$, is

(a) 36 (b) 40 (c) 44 (d) 52

**6.** A person draws out two balls successively from a bag containing 6 red and 4 white balls. The probability that at least one of them will be red is

(a) $\frac{78}{90}$ (b) $\frac{30}{90}$
(c) $\frac{48}{90}$ (d) $\frac{12}{90}$

**7.** If $u(x)$ and $v(x)$ be differentiable functions for $x>0$ such that $xu'+v=0$, $xv'+u=0$, $u(1)=0$, $v(1)=2$, then $\int_0^1\frac{u}{v}dx=$

(a) 1 (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}-1$ (d) $\frac{\pi}{4}-1$

**8.** If $f(x)$ is a polynomial such that $f(x)\,f(y)=f(xy)-f(x)-f(y)\ \forall x,\ y$ and $f(2)=7$, then $f(-2)=$

(a) 8 (b) 10 (c) –7 (d) –9

**9.** Let $P(x)=a_0+a_1x+a_2x^2+\ldots+a_nx^n$.
If $|P(x)|\le|e^x-1-1|\ \forall\ x\ge 0$, then the maximum value of $|a_1+2a_2+3a_3+\ldots..+na_n|$ is

(a) $\frac{1}{n}$ (b) 1 (c) $n$ (d) $e$

**10.** If $A>0$, $B>0$ and $A+B=\frac{\pi}{6}$, then the minimum value of $\tan A+\tan B$ is

(a) $\sqrt{3}-\sqrt{2}$ (b) $4-2\sqrt{3}$
(c) $\frac{2}{\sqrt{3}}$ (d) $2-\sqrt{3}$

**11.** If the equation $x^4+4x^3-2x^2-12x+k=0$ has 4 real roots , then

(a) $-1<k<10$ (b) $-7<k<7$
(c) $-7<k<9$ (d) $1<k<9$

**12.** The number of ways of distributing 20 identical marbles among three children so that each gets atleast one and no two get the same number is

(a) 118 (b) 120 (c) 220 (d) 144

**13.** If $\lim\limits_{x\to 0}\frac{a\sin x-bx+cx^2+x^3}{2x^2\ln(1+x)-2x^3+x^4}=L$, where $L$ is finite and non-zero, then $L$ is equal to

(a) $\frac{1}{20}$ (b) $\frac{3}{20}$ (c) $\frac{3}{40}$ (d) $\frac{5}{41}$

**14.** If $f: R \to S$, defined by $f(x) = \sin x - \sqrt{3}\cos x + 1$, is onto, then the interval of $S$ is

(a) $[-1, 3]$ (b) $[-1, 1]$ (c) $[0, 1]$ (d) $[0, 3]$

**15.** Let $A$ and $B$ be two matrices such that $A = [a_{ij}]_{2\times 3}$

$$= \begin{bmatrix} x & 1 & 2 \\ 3x & -1 & \dfrac{1}{4x^2+1} \end{bmatrix} \text{ and } B = [b_{ij}]_{3\times 2} = \begin{bmatrix} \dfrac{1}{x^2} & \dfrac{1}{x} \\ 2x & 2 \\ 3 & x \end{bmatrix},$$

where $x > 0$. If matrix $C$ is defined as $C = [c_{ij}]_{2\times 2}$, where $c_{ij} = \sum_{r=1}^{3} a_{ir} b_{rj}\ \forall 1 \le i, j \le 2$, then the minimum value of $\Delta(x) = \sum_{1\le i\le j\le 2} c_{ij}$ is

(a) $\dfrac{57}{4}$ (b) $\dfrac{57}{2}$ (c) $\dfrac{55}{4}$ (d) $\dfrac{55}{2}$

**16.** The number of roots of the equation $x^2 = \cot x$ in $[0, 2\pi]$ is

(a) 0 (b) 1 (c) 2 (d) 3

**17.** If $\int \dfrac{dx}{x^n(1+x^n)^{\frac{1}{n}}} = a\left(1+\dfrac{1}{x^n}\right)^b + c$, then $ab$ equals

(a) 1 (b) $-1$ (c) $\dfrac{1}{n}$ (d) $-\dfrac{1}{n}$

**18.** A line $L$ has intercepts $a$ and $b$ on the coordinate axes. When the axes are rotated through a given angle keeping the origin fixed, the same line $L$ has intercepts $p$ and $q$. Then

(a) $a^2 + b^2 = p^2 + q^2$ (b) $\dfrac{1}{a^2} + \dfrac{1}{b^2} = \dfrac{1}{p^2} + \dfrac{1}{q^2}$

(c) $a^2 + p^2 = b^2 + q^2$ (d) $\dfrac{1}{a^2} + \dfrac{1}{p^2} = \dfrac{1}{b^2} + \dfrac{1}{q^2}$

**19.** If two distinct chords, drawn from the point $(p, q)$ on the circle $x^2 + y^2 = px + qy$, where $pq \neq 0$, are bisected by the $x$-axis, then

(a) $p^2 = q^2$ (b) $p^2 = 8q^2$

(c) $p^2 < 8q^2$ (d) $p^2 > 8q^2$

**20.** If $[\sin^{-1}\cos^{-1}\sin^{-1}\tan^{-1} x] = 1$, where $[.]$ denotes the greatest integer function, then $x$ belongs to the interval

(a) $[\tan\sin\cos 1, \tan\sin\cos\sin 1]$

(b) $[\sin\cos\tan 1, \sin\cos\sin\tan 1]$

(c) $[1, -1]$

(d) none of these

## NUMERICAL VALUE TYPE

**21.** If $\vec{u}, \vec{v}, \vec{w}$, are three non-coplanar unit vectors and $\alpha$, $\beta$, $\gamma$ are the angles between $\vec{u}$ and $\vec{v}$, $\vec{v}$ and $\vec{w}$, $\vec{w}$ and $\vec{u}$, respectively, and $\vec{x}, \vec{y}, \vec{z}$ are unit vectors along the bisectors of the angles $\alpha$, $\beta$, $\gamma$, respectively. If $[\vec{x}\times\vec{y}\ \ \vec{y}\times\vec{z}\ \ \vec{z}\times x] = \dfrac{1}{\lambda}[\vec{u}\,\vec{v}\,\vec{w}]^2 \sec^2\dfrac{\alpha}{2}\sec^2\dfrac{\beta}{2}\sec^2\dfrac{\gamma}{2}$, then the value of $\sqrt{\lambda}$ is _____.

**22.** If $f(x)$ and $g(x)$ are any two real valued functions such that $|f(x) + g(x)| \ge |f(x)| + |g(x)|$ and $g(x) \neq 0$, $f(x)\cdot g(x) \le 0$ then the value of $\sum_{r=1}^{100} f(r) =$ _____.

**23.** If $\cot^{-1}\left(\dfrac{n^2 - 10n + 21.6}{\pi}\right) > \dfrac{\pi}{6}, n \in N$, then the minimum value of $n$ is _____.

**24.** The eccentricity of an ellipse whose axes are the coordinate axes is $\dfrac{3}{5}$. If the length of its latus rectum is $\dfrac{32}{5}$, then the maximum distance of its centre from a normal is _____.

**25.** If the foot of perpendicular from the point $P(4, 6, 2)$ to the line $x - y - z + 2 = 0 = 2x - y - 4z + 6$ is $(a, b, c)$, then $\dfrac{a+b+c}{3}$ is _____.

**26.** Let $f(x + y) = f(x) + f(y) - 2xy - 1$ for all $x$ and $y$. If $f'(0)$ exists and $f'(0) = -\sin\alpha$, then the value of $f\{f'(0)\}$ is _____.

**27.** A line is drawn through the point $A(3, 4)$ inclined at an acute angle with positive direction of the $x$-axis. If it cuts the circle $x^2 + y^2 = 4$ in $B$ and $C$, then the maximum value of $\dfrac{|AB + AC|}{2}$ is _____.

**28.** Each of two boxes contains both black and white marbles, and the total number of marbles in the two boxes is 25. One marble is taken out of each box. The probability that both marbles are black is

**Recipe for Success**

"Arise, awake and stop not till the goal is reached"

– Swami Vivekananda

$\frac{27}{50}$. If the probability that both marbles are white is $\frac{m}{n}$, where $m$, $n$, are relatively prime positive integers, then $m$ is _____.

**29.** If $\lim\limits_{x\to 0}\dfrac{\sin(3x+a)-3\sin(2x+a)+3\sin(x+a)-\sin a}{x^3} = -\cos 1$, then $a$ = _____.

**30.** Let $k$ be a positive real number and

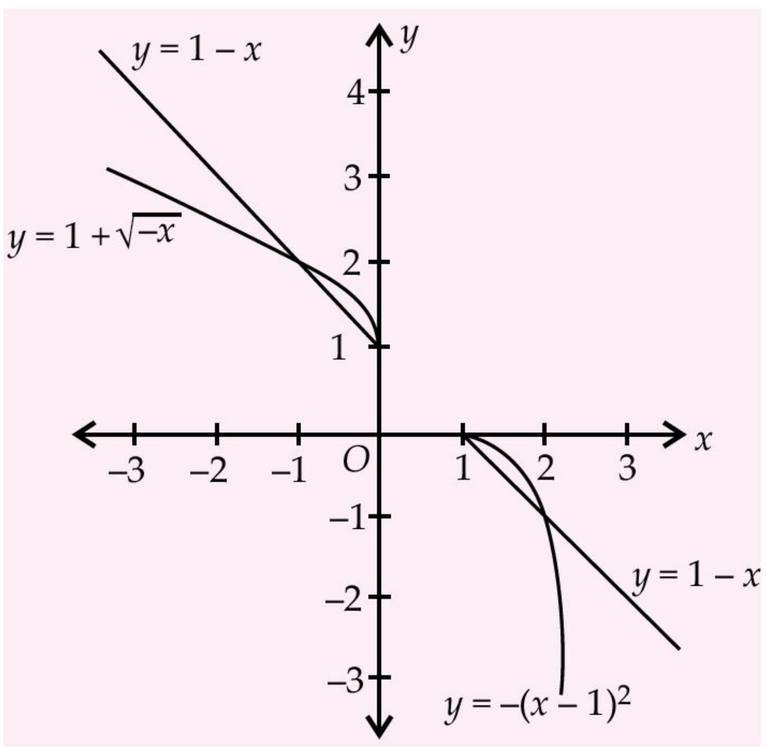
$$A=\begin{bmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1 & -2k \\ -2\sqrt{k} & 2k & -1 \end{bmatrix},\ B=\begin{bmatrix} 0 & 2k-1 & \sqrt{k} \\ 1-2k & 0 & 2\sqrt{k} \\ -\sqrt{k} & -2\sqrt{k} & 0 \end{bmatrix}$$

If $\det(\text{adj } A) + \det(\text{adj } B) = 10^6$, then $[k]$ is _____.
(where $[\cdot]$ denotes the greatest integer function)

## SOLUTIONS

**1. (d) :** $f^{-1}(x)=\begin{cases} -x+1, & x\ge 1 \\ 1+\sqrt{-x}, & x\le 0 \end{cases}$

the solutions of $f(x) - f^{-1}(x) = 0$ are $x = -1, 1, 0, 2$.




**2. (b) :** $121^n - 25^n = (96 + 25)^n - 25^n$ is divisible by 96.
$1900^n - (-4)^n = (1904 - 4)^n - (-4)^n$ is divisible by 1904.
Hence, both the above are divisible by 16.
Further, $121^n - (-4)^n$ is divisible by 125 and $1900^n - 25^n$ is divisible by 1875. Hence, both the above are divisible by 125.
$\therefore$ The given number is divisible by $16 \times 125 = 2000$.

**3. (b) :** $f(x) - x = 0$ has imaginary roots as $\alpha$ and $\beta$, so,
$f(x) - x \neq 0\ \forall\ x \in R$
$\Rightarrow f(x) - x > 0$ or $f(x) - x < 0\ \forall\ x \in R$ ...(i)
$\Rightarrow f(f(x)) - f(x) > 0$
or $f(f(x)) - f(x) < 0\ \forall\ x \in R$ ...(ii)
Adding, (i) and (ii), we get
$f(f(x)) - x > 0$ or $f(f(x)) - x < 0\ \forall\ x \in R$
$\Rightarrow$ Roots of $f(f(x)) - x = 0$

Let $z=\begin{vmatrix} 2 & \alpha & \delta \\ \beta & 0 & \alpha \\ \gamma & \beta & 1 \end{vmatrix} \Rightarrow \overline{z}=\begin{vmatrix} 2 & \overline{\alpha} & \overline{\delta} \\ \overline{\beta} & 0 & \overline{\alpha} \\ \overline{\gamma} & \overline{\beta} & 1 \end{vmatrix}=\begin{vmatrix} 2 & \beta & \gamma \\ \alpha & 0 & \beta \\ \delta & \alpha & 1 \end{vmatrix}=z$

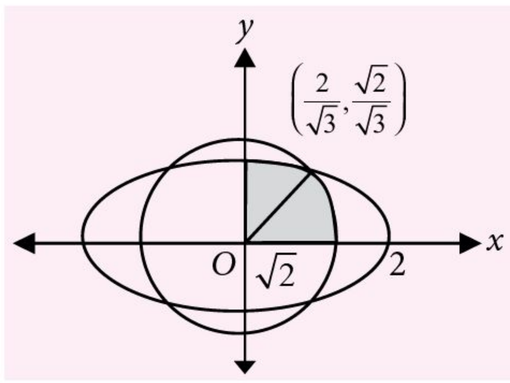
$\therefore$ $z$ is purely real.

**4. (d) :** Eliminating $y$ from given equations, we have,

$x^2 + 4(2 - x^2) = 4 \Rightarrow x = \dfrac{2}{\sqrt{3}}$

The shaded area, $I=\int_0^{\frac{2}{\sqrt{3}}} y(\text{ellipse})\,dx + \int_{\frac{2}{\sqrt{3}}}^{\sqrt{2}} y(\text{circle})\,dx$

$=\frac{1}{2}\int_0^{\frac{2}{\sqrt{3}}}\sqrt{4-x^2}\,dx + \int_{\frac{2}{\sqrt{3}}}^{\sqrt{2}}\sqrt{2-x^2}\,dx$




$=\left|\frac{1}{4}x\sqrt{4-x^2}+\sin^{-1}\frac{x}{2}\right|_0^{\frac{2}{\sqrt{3}}}$

$+\left|\frac{x}{2}\sqrt{2-x^2}+\sin^{-1}\frac{x}{\sqrt{2}}\right|_{\frac{2}{\sqrt{3}}}^{\sqrt{2}}$

$=\frac{1}{2\sqrt{3}}\sqrt{4-\frac{4}{3}}-\frac{1}{\sqrt{3}}\sqrt{2-\frac{4}{3}}+\sin^{-1}\frac{1}{\sqrt{3}}+\frac{\pi}{2}-\sin^{-1}\frac{\sqrt{2}}{\sqrt{3}}$

$=\sin^{-1}\frac{1}{\sqrt{3}}+\cos^{-1}\sqrt{\frac{2}{3}}=2\tan^{-1}\frac{1}{\sqrt{2}}=\tan^{-1}2\sqrt{2}$

$\therefore$ Required area $= 4I = 4\tan^{-1}2\sqrt{2}$

**5. (a) :** $y + z = 10 - 2x$
The number of solutions is

$$\sum_{x=0}^{5}\binom{10-2x+2-1}{1}=\sum_{x=0}^{5}(11-2x)=66-2\cdot\frac{5\times 6}{2}=36$$

**6. (a) :** Let S is sample space.
$\therefore$ $n(S) = {}^{10}C_1 \times {}^{9}C_1 = 90$
Let $E_1$ be the event that I ball drawn is red and II ball drawn is white.
$\therefore$ $n(E_1) = {}^{6}C_1 \times {}^{4}C_1 = 24$
Let $E_2$ be the event that I ball drawn is white and II ball drawn is red.
$\therefore$ $n(E_2) = {}^{4}C_1 \times {}^{6}C_1 = 24$
Let $E_3$ be the event that both balls drawn are red
$\therefore$ $n(E_3) = {}^{6}C_1 \times {}^{5}C_1 = 30$
$\because$ $E_1$, $E_2$ and $E_3$ are mutually exclusive.
$\therefore$ $n(E_1 \cup E_2 \cup E_3) = n(E_1) + n(E_2) + n(E_3)$
$= 24 + 24 + 30 = 78$

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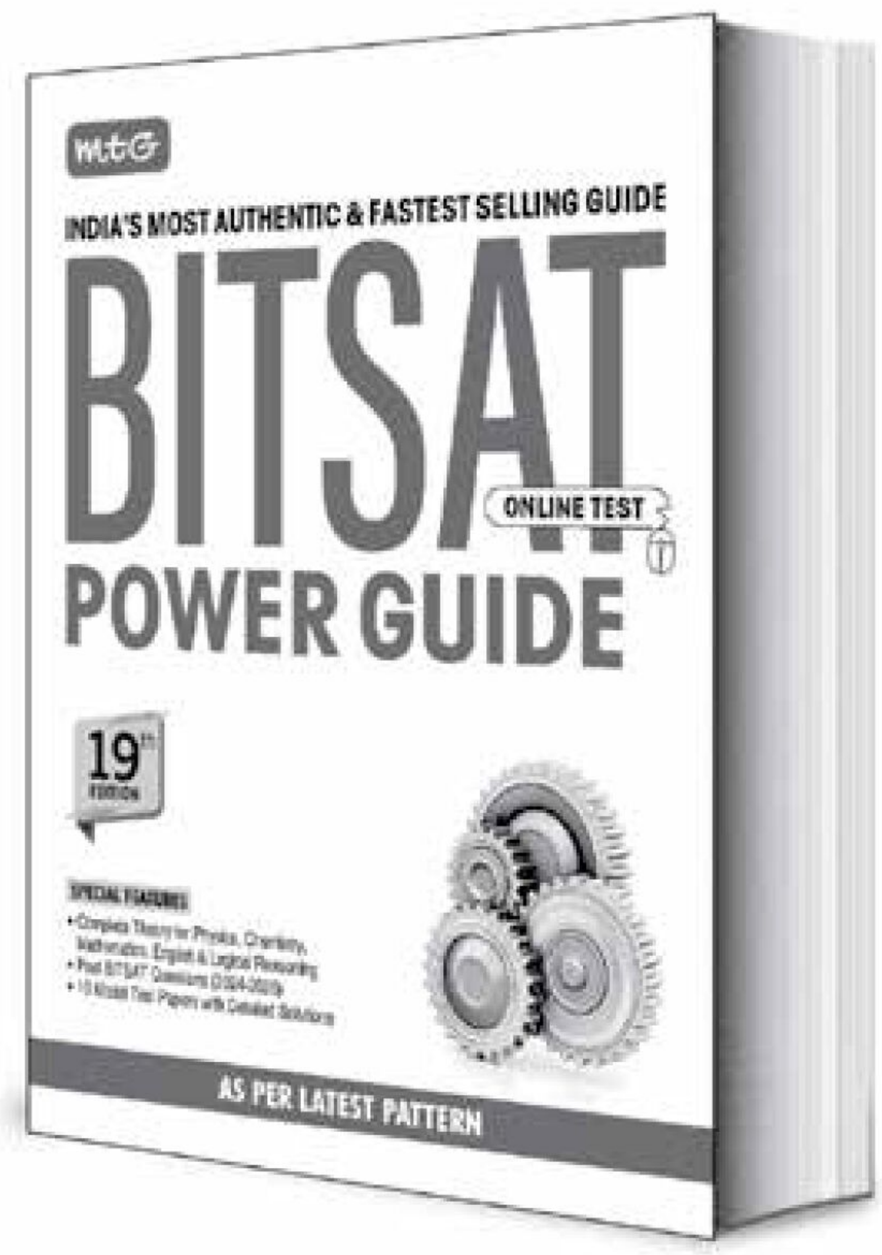


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$\therefore$ Probability that at least one of them will be red

$$= \frac{n(E_1 \cup E_2 \cup E_3)}{n(S)} = \frac{78}{90}$$

**7. (c) :** $xu' + v = 0$ ...(i) and $xv' + u = 0$ ...(ii)

(i) + (ii) $\Rightarrow x\frac{d}{dx}(u+v) + u + v = 0$

$\Rightarrow x(u+v) = A$

When $x = 1, u = 0, v = 2 \Rightarrow A = 2$

$\Rightarrow u + v = \frac{2}{x}$ ...(iii)

(i) – (ii) $\Rightarrow x\frac{d}{dx}(u-v) = u - v$

$\Rightarrow \int\frac{d(u-v)}{u-v} = \int\frac{dx}{x}$

$\Rightarrow \ln(u - v) = \ln x + \ln B$ or $u - v = Bx$

When $x = 1, u = 0, v = 2$

$\Rightarrow B = -2 \Rightarrow u - v = -2$ ...(iv)

Solving (iii) and (iv), we get $u = \frac{1}{x} - x, v = \frac{1}{x} + x$

$$\int_0^1 \frac{u}{v}dx = \int_0^1 \frac{1-x^2}{1+x^2}dx = \int_0^1\left(\frac{2}{1+x^2} - 1\right)dx = \frac{\pi}{2} - 1$$

**8. (d) :** The given equation can be rewritten as

$\Rightarrow (f(x) + 1)(f(y) + 1) = f(xy) + 1$

If $g(x) = f(x) + 1$, then $g(x)\, g(y) = g(xy)$

$\Rightarrow g(x) = x^n \Rightarrow f(x) = x^n - 1$

Now, $f(2) = 7 \Rightarrow 2^n - 1 = 7 \Rightarrow n = 3$

$\Rightarrow f(x) = x^3 - 1 \Rightarrow f(-2) = -9$

**9. (b) :** $|P(1)| \le |e^{1-1} - 1| = 0 \Rightarrow P(1) = 0$

$\therefore |a_1 + 2a_2 + 3a_3 + \ldots + na_n| = |P'(1)|$

$$\Rightarrow \left|\lim_{h\to 0}\frac{P(1+h) - P(1)}{h}\right| \le \left|\lim_{h\to 0}\frac{e^h - 1}{h}\right| = |1| = 1$$

**10. (b) :** We know that, $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{y}{1 - \tan A \tan B}$, where $y = \tan A + \tan B$

$\Rightarrow \tan A \tan B = 1 - \sqrt{3}y$

Also A.M. $\ge$ G.M.

$\Rightarrow \frac{\tan A + \tan B}{2} \ge \sqrt{\tan A \tan B}$

$\Rightarrow y \ge 2\sqrt{1 - \sqrt{3}y} \Rightarrow y^2 \ge 4 - 4\sqrt{3}y$

$\Rightarrow y^2 + 4\sqrt{3}y - 4 \ge 0$

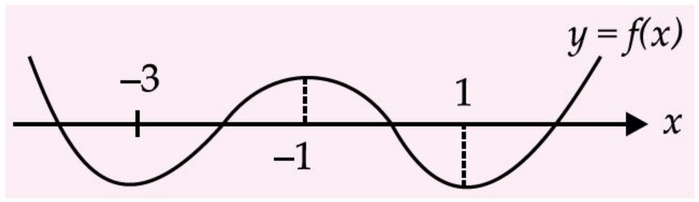
$\Rightarrow y \le -2\sqrt{3} - 4$ or $y \ge -2\sqrt{3} + 4$

($y \le -2\sqrt{3} - 4$ is not possible as $\tan A, \tan B > 0$)

**11. (c) :** $f(x) = x^4 + 4x^3 - 2x^2 - 12x + k$

$\Rightarrow f'(x) = 4(x^3 + 3x^2 - x - 3) = 4(x+3)(x+1)(x-1)$

Now, $f'(x) = 0 \Rightarrow x = -3, -1, 1$




Now, $f(-3) < 0 \Rightarrow -9 + k < 0 \Rightarrow k < 9$;

$f(-1) > 0 \Rightarrow 7 + k > 0 \Rightarrow k > -7$;

$f(1) < 0 \Rightarrow -9 + k < 0 \Rightarrow k < 9 \therefore -7 < k < 9$

**12. (d) :** $a < b < c, a + b + c = 20$

| | | |
|---|---|---|
| $a = 1 \Rightarrow (b, c) = (2, 17), (3, 16), \ldots (9, 10)$ | : | 8 pairs |
| $a = 2 \Rightarrow (b, c) = (3, 15), (4, 14), \ldots (8, 10)$ | : | 6 pairs |
| $a = 3 \Rightarrow (b, c) = (4, 13), (5, 12), \ldots (8, 9)$ | : | 5 pairs |
| $a = 4 \Rightarrow (b, c) = (5, 11), (6, 10), (7, 9)$ | : | 3 pairs |
| $a = 5 \Rightarrow (b, c) = (6, 9), (7, 8)$ | : | 2 pairs |
| | | Total 24 |

$\therefore$ There are 24 triples $(a, b, c)$

Permuting them, the desired number of ways is $24 \times 6 = 144$.

**13. (c) :** By series,

$$\lim_{x\to 0}\frac{a\left(x - \frac{x^3}{6} + \frac{x^5}{120} - \ldots\right) - bx + cx^2 + x^3}{2x^2\left(x - \frac{x^2}{2} + \frac{x^3}{3}\ldots\right) - 2x^3 + x^4}$$

$$= \lim_{x\to 0}\frac{(a-b)x + cx^2 + \left(1 - \frac{a}{6}\right)x^3 + \frac{a}{120}x^5 + \ldots}{\frac{2}{3}x^5 + \ldots} = L$$

$\Rightarrow a = b, c = 0, 1 - \frac{a}{6} = 0, a = 6$

$\therefore L = \frac{a}{120} \times \frac{3}{2} = \frac{6}{120} \times \frac{3}{2} = \frac{3}{40}$

**14. (a) :** If $f: R \to S$ is onto, then range of $f$ is equal to its co-domain *i.e.*, $S$ = Range $(f)$

Now, $f(x) = \sin x - \sqrt{3}\cos x + 1$

$\Rightarrow f(x) = 2\left(\frac{1}{2}\sin x - \frac{\sqrt{3}}{2}\cos x\right) + 1 = 2\sin\left(x - \frac{\pi}{3}\right) + 1$

We know that, $-1 \le \sin\left(x - \frac{\pi}{3}\right) \le 1$ for all $x \in$ R

$\Rightarrow -1 \le 2\sin\left(x - \frac{\pi}{3}\right) + 1 \le 3$ for all $x \in R$

$\Rightarrow -1 \le f(x) \le 3$ for all $x \in R$

$\Rightarrow$ Range $(f) = [-1, 3]$. Hence, $S = [-1, 3]$

**15. (a) :** $C = AB = \begin{bmatrix} x & 1 & 2 \\ 3x & -1 & \dfrac{1}{4x^2+1} \end{bmatrix} \begin{bmatrix} \dfrac{1}{x^2} & \dfrac{1}{x} \\ 2x & 2 \\ 3 & x \end{bmatrix}$

$\therefore \quad \Delta(x) = \sum_{1 \le i \le j \le 2} c_{ij} = c_{11} + c_{12} + c_{22}$

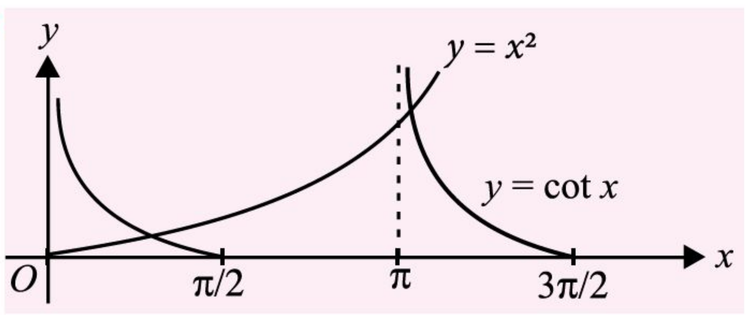
$= \dfrac{1}{x} + 2x + 6 + 1 + 2 + 2x + 3 - 2 + \dfrac{x}{4x^2+1}$

$= \dfrac{1}{x} + 4x + 10 + \dfrac{1}{4x + \dfrac{1}{x}}$

Let $\dfrac{1}{x} + 4x = t \Rightarrow t \ge 4$

$\therefore \quad |\Delta(x)|_{\min} = 4 + 10 + \dfrac{1}{4} = \dfrac{57}{4}$

**16. (c) :**




The two curves $y = x^2$ and $y = \cot x$ intersect at two points, one for $0 < x < \dfrac{\pi}{2}$ and the other one for $\dfrac{3\pi}{2} < x < \pi$.

**17. (d) :** $\displaystyle\int \frac{dx}{x^n (1+x^n)^{\frac{1}{n}}} = \int \frac{dx}{x^{n+1}\left(\frac{1}{x^n}+1\right)^{\frac{1}{n}}}$

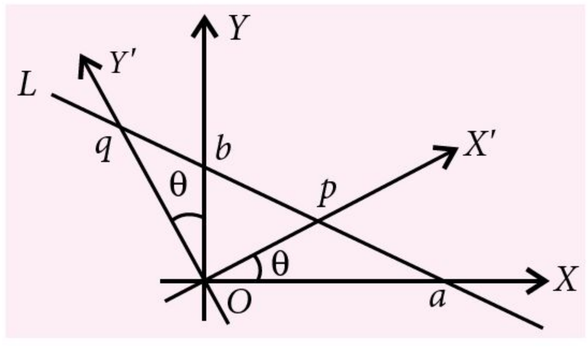
Put $t = \dfrac{1}{x^n} + 1 \Rightarrow dt = \dfrac{-n}{x^{n+1}} dx$

$= -\dfrac{1}{n}\int t^{-\frac{1}{n}} dt = \dfrac{1}{1-n} t^{1-\frac{1}{n}} + c = \dfrac{1}{1-n}\left(\dfrac{1}{x^n}+1\right)^{\frac{n-1}{n}} + c$

$\therefore$ By comparing, we get $ab = \left(\dfrac{1}{1-n}\right)\left(\dfrac{n-1}{n}\right) = -\dfrac{1}{n}$

**18. (b) :** Let $OX'$, $OY'$ be the new axes obtained by rotation of old axes $OX$ and $OY$.

Equation of $L$ in $x, y$ system is $\dfrac{x}{a} + \dfrac{y}{b} = 1$

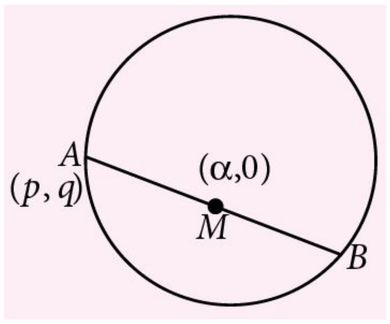



Equation of $L$ in $X, Y$ system is $\dfrac{X}{p} + \dfrac{Y}{q} = 1$

The distances of $O$ from the lines are equal.

$\therefore \quad \dfrac{1}{\sqrt{\dfrac{1}{a^2} + \dfrac{1}{b^2}}} = \dfrac{1}{\sqrt{\dfrac{1}{p^2} + \dfrac{1}{q^2}}} \Rightarrow \dfrac{1}{a^2} + \dfrac{1}{b^2} = \dfrac{1}{p^2} + \dfrac{1}{q^2}$

**19. (d) :** $M(\alpha, 0)$ is on the $x$-axis.
$M$ is the midpoint of $AB$.




$\therefore \quad B \equiv (2\alpha - p, -q)$
which lies on the circle
$x^2 + y^2 = px + qy$
$\therefore \quad (2\alpha - p)^2 + q^2 = p(2\alpha - p) - q^2,$
$\Rightarrow \quad 2\alpha^2 - 3p\alpha + p^2 + q^2 = 0$
As these are two chords, there are two values of $\alpha$.
$\therefore$ Discriminant $> 0$
$\Rightarrow \quad 9p^2 - 8(p^2 + q^2) > 0$
$\Rightarrow \quad p^2 > 8q^2$

**20. (a) :** We have, $[\sin^{-1} \cos^{-1} \sin^{-1} \tan^{-1} x] = 1$

$\Rightarrow \quad 1 \le \sin^{-1} \cos^{-1} \sin^{-1} \tan^{-1} x \le \dfrac{\pi}{2}$

$\Rightarrow \quad \sin 1 \le \cos^{-1} \sin^{-1} \tan^{-1} x \le 1$

$\Rightarrow \quad \cos \sin 1 \ge \sin^{-1} \tan^{-1} x \ge \cos 1$

$\Rightarrow \quad \sin \cos \sin 1 \ge \tan^{-1} x \ge \sin \cos 1$

$\Rightarrow \quad \tan \sin \cos \sin 1 \ge x \ge \tan \sin \cos 1$

Hence, $x \in [\tan \sin \cos 1, \tan \sin \cos \sin 1]$

**21. (4) :** We know that $[\vec{x} \times \vec{y} \quad \vec{y} \times \vec{z} \quad \vec{z} \times \vec{x}] = [\vec{x}\,\vec{y}\,\vec{z}]^2$
Also a vector along the bisector of given two unit vectors $\vec{u}, \vec{v}$ is $\vec{u} + \vec{v}$.

A unit vector along the bisector is $\dfrac{\vec{u} + \vec{v}}{|\vec{u} + \vec{v}|}$.

$|\vec{u} + \vec{v}|^2 = 1 + 1 + 2\vec{u} \cdot \vec{v} = 2 + 2\cos\alpha = 4\cos^2\left(\dfrac{\alpha}{2}\right)$

$\Rightarrow \vec{x} = \dfrac{\vec{u} + \vec{v}}{2\cos\left(\dfrac{\alpha}{2}\right)}$

Similarly, $\vec{y} = \dfrac{\vec{v} + \vec{w}}{2\cos(\beta/2)}$ and $\vec{z} = \dfrac{\vec{u} + \vec{w}}{2\cos(\gamma/2)}$

$\therefore [\vec{x}\,\vec{y}\,\vec{z}] = \dfrac{1}{8}[\vec{u} + \vec{v} \quad \vec{v} + \vec{w} \quad \vec{u} + \vec{w}] \times \sec\left(\dfrac{\alpha}{2}\right)\sec\left(\dfrac{\beta}{2}\right)\sec\left(\dfrac{\gamma}{2}\right)$

$= \dfrac{1}{8} \times 2[\vec{u}\,\vec{v}\,\vec{w}] \sec\left(\dfrac{\alpha}{2}\right)\sec\left(\dfrac{\beta}{2}\right)\sec\left(\dfrac{\gamma}{2}\right)$

$$= \frac{1}{4}[\vec{u}\,\vec{v}\,\vec{w}]\sec\left(\frac{\alpha}{2}\right)\sec\left(\frac{\beta}{2}\right)\sec\left(\frac{\gamma}{2}\right)$$

Now, $[\vec{x}\times\vec{y}\quad \vec{y}\times\vec{z}\quad \vec{z}\times\vec{x}] = [\vec{x}\,\vec{y}\,\vec{z}]^2$

$$= \frac{1}{16}[\vec{u}\,\vec{v}\,\vec{w}]^2\sec^2\left(\frac{\alpha}{2}\right)\sec^2\left(\frac{\beta}{2}\right)\sec^2\left(\frac{\gamma}{2}\right)$$

So, $\sqrt{\lambda} = \sqrt{16} \Rightarrow \lambda = 4$

**22. (0) :** We know that, $|f(x)+g(x)| \le |f(x)| + |g(x)|$

But given, $|f(x)+g(x)| \ge |f(x)| + |g(x)|$

$\therefore \quad |f(x)+g(x)| = |f(x)| + |g(x)|$

$\therefore \quad f(x)\cdot g(x) \ge 0$

But given that $f(x)\cdot g(x) \le 0$

$\Rightarrow \quad f(x)\times g(x) = 0 \Rightarrow f(x) = 0 \qquad (\because\ g(x) \ne 0)$

$\therefore \quad \sum_{r=1}^{100} f(r) = 0$

**23. (3) :** $\cot^{-1}\left(\frac{n^2-10n+21.6}{\pi}\right) > \frac{\pi}{6}$

$$\Rightarrow \quad \frac{n^2-10n+21.6}{\pi} < \cot\frac{\pi}{6}$$

$[\because \cot x \text{ is decreasing function for } 0 < x < \pi]$

$$\Rightarrow \quad n^2-10n+21.6 < \pi\sqrt{3} \Rightarrow (n-5)^2 + 21.6 - 25 < \sqrt{3}\pi$$

$$\Rightarrow \quad (n-5)^2 < \sqrt{3}\,\pi + 3.4$$

$$\Rightarrow \quad 5-\sqrt{\sqrt{3}\pi+3.4} < n < 5+\sqrt{\sqrt{3}\pi+3.4}$$

But $\sqrt{3}\pi = 5.5$ nearly

$\therefore \quad \sqrt{\sqrt{3}\pi+3.4} = \sqrt{8.9} = 2.9$ nearly

$\therefore \quad 2.1 < n < 7.9 \quad \therefore \quad n = 3, 4, 5, 6, 7 \qquad [\because\ n \in N]$

Hence, minimum value of $n$ is 3.

**24. (1) :** Length of latus rectum

$$= 2a(1-e^2) = \frac{32}{5} \Rightarrow a\left(1-\frac{9}{25}\right) = \frac{16}{5}$$

$$\therefore \quad a = 5,\ b = a\sqrt{1-e^2} = 5\sqrt{1-\frac{9}{25}} = 4$$

So, the ellipse is $\frac{x^2}{25} + \frac{y^2}{16} = 1$

The normal at $(5\cos\theta, 4\sin\theta)$ is

$$\frac{5x}{\cos\theta} - \frac{4y}{\sin\theta} = 5^2 - 4^2 = 9$$

The distance of the centre from normal is

$$d = \frac{9}{\sqrt{\frac{25}{\cos^2\theta} + \frac{16}{\sin^2\theta}}} = \frac{9}{\sqrt{25\sec^2\theta + 16\,\mathrm{cosec}^2\theta}}$$

$$= \frac{9}{\sqrt{41 + 25\tan^2\theta + 16\cot^2\theta}}$$

As $25\tan^2\theta + 16\cot^2\theta \ge 2\cdot 5\cdot 4 = 40$

(By A.M.-G.M. inequality)

$$\therefore \quad d \le \frac{9}{\sqrt{41+40}} = 1$$

**25. (4) :** The d.r's of the line are given by

$l - m - n = 0,\ 2l - m - 4n = 0$

$\Rightarrow \quad l : m : n = 3 : 2 : 1$

A point on the line with $z = 0$ is given by

$x - y + 2 = 0,\ 2x - y + 6 = 0$

$\Rightarrow \quad (x, y, z) = (-4, -2, 0)$

The line is $\frac{x+4}{3} = \frac{y+2}{2} = \frac{z}{1} = k$

So, $Q(3k-4, 2k-2, k)$ is a point on the line.

The d.r's of $PQ$ are $3k-8, 2k-8, k-2$

[Given $P \equiv (4, 6, 2)$]

Now, $PQ$ is perpendicular to the line.

$\therefore \quad 3(3k-8) + 2(2k-8) + k - 2 = 0$

$\Rightarrow \quad k = 3$

Thus, the foot is $Q(5, 4, 3)$

So, $\frac{a+b+c}{3} = \frac{5+4+3}{3} = 4$

**26. (1) :** $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$$= \lim_{h\to 0}\frac{\{f(x)+f(h)-2xh-1\}-f(x)}{h}$$

(Using the given relation)

$$= \lim_{h\to 0}(-2x) + \lim_{h\to 0}\left(\frac{f(h)-1}{h}\right)$$

$$= \lim_{h\to 0}(-2x) + \lim_{h\to 0}\left(\frac{f(h)-f(0)}{h}\right)$$

[Putting $x = 0 = y$ in the given relation, we can find $f(0) = f(0) + f(0) + 0 - 1 \quad \Rightarrow \quad f(0) = 1$]

$\therefore \quad f'(x) = -2x + f'(0)$

$\Rightarrow \quad f(x) = -x^2 - \sin\alpha\cdot x + C$

$\Rightarrow \quad f(0) = -0 - 0 + C \quad \Rightarrow C = 1$

$\therefore \quad f(x) = -x^2 - \sin\alpha\cdot x + 1$

So, $f\{f'(0)\} = f(-\sin\alpha) = -\sin^2\alpha + \sin^2\alpha + 1$

$\therefore \quad f\{f'(0)\} = 1$

**27. (5) :** Equation of the line passing through $A(3, 4)$ and inclined at an acute angle $\theta$ is

$$\frac{x-3}{\cos\theta}=\frac{y-4}{\sin\theta}=r=AB \text{ or } AC$$

$\Rightarrow \quad x = 3 + r\cos\theta,\ y = 4 + r\sin\theta$

Since $(x, y)$ lies on $x^2 + y^2 = 4$, we have

$(3 + r\cos\theta)^2 + (4 + r\sin\theta)^2 = 4$

$\Rightarrow \quad r^2 + 2(3\cos\theta + 4\sin\theta)r + 21 = 0$

$\Rightarrow \quad AB + AC = -2(3\cos\theta + 4\sin\theta)$

$$\le 2\sqrt{(3)^2+(4)^2}$$

$\therefore$ Maximum value of $\dfrac{|AB+AC|}{2}$ is 5.

**28. (1) :**

| Box I | Box II |
|---|---|
| $a$ black marbles | $c$ black marbles |
| $b$ white marbles | $d$ white marbles |

Given, $a + b + c + d = 25$ and $\dfrac{a}{a+b}\cdot\dfrac{c}{c+d}=\dfrac{27}{50}$

$27(a + b)(c + d) = 50\ ac$, which means $ac$ is divisible by 27.

So, $ac = 27\ k$

$\Rightarrow \quad (a + b)(c + d) = 50\ k,$

$\Rightarrow \quad (a + b)(25 - (a + b)) = 50\ k$

If $k = 3 \Rightarrow a + b = 15,\ c + d = 10,\ a = 9,\ b = 6,\ c = 9,\ d = 1$

If $k = 2 \Rightarrow a + b = 20,\ c + d = 5,\ a = 18,\ b = 2,\ c = 3,\ d = 2$

In either case, the probability of two white marbles is

$$\frac{bd}{(a+b)(c+d)}=\frac{6}{150}=\frac{4}{100}=\frac{1}{25}=\frac{m}{n}$$

$\therefore \quad m = 1$

**29. (1) :** $\lim\limits_{x\to0}\dfrac{\sin(3x+a)-3\sin(2x+a)+3\sin(x+a)-\sin a}{x^3}$

$$=\lim_{x\to0}\frac{2\sin\frac{3x}{2}\cos\left(\frac{3x+2a}{2}\right)-3\left(2\sin\frac{x}{2}\cos\frac{3x+2a}{2}\right)}{x^3}$$

$$=\lim_{x\to0}\frac{2\cos\left(\frac{3x+2a}{2}\right)\left[3\sin\frac{x}{2}-4\sin^3\frac{x}{2}-3\sin\frac{x}{2}\right]}{x^3}$$

$$=\lim_{x\to0}2\cos\left(\frac{3x+2a}{2}\right)\left\{-4\frac{\sin^3(x/2)}{x^3}\right\}$$

$$=2\cos a\times\left(-\frac{4}{8}\right)=-\cos a=-\cos 1$$

$\therefore \quad a = 1$

**30. (4) :** $\det A=(2k-1)(4k^2-1)+2\sqrt{k}\ (4k\sqrt{k}+2\sqrt{k})+2\sqrt{k}\ (4k\sqrt{k}+2\sqrt{k})$

$= (2k - 1)(4k^2 - 1) + 8(2k + 1)k$

$= (2k + 1)((2k - 1)^2 + 8k) = (2k + 1)^3$

$\therefore \quad \det(\text{adj } A) = (\det A)^2 = (2k + 1)^6$

And $\det B = 0$, since $B$ is skew symmetric matrix of order 3.

$\therefore \quad \det(\text{adj } B) = (\det B)^2 = 0$

Now, $\det(\text{adj } A) + \det(\text{adj } B) = 10^6$ [Given]

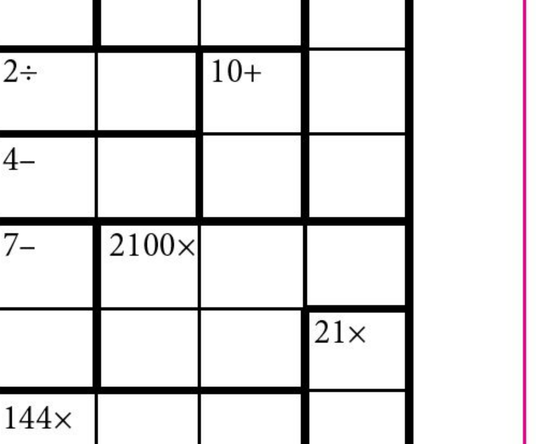
$\therefore \quad (2k + 1)^6 = 10^6$

$\Rightarrow \quad 2k + 1 = 10$

$\Rightarrow \quad k = 4.5$

So, $[k] = [4.5] = 4$

# PUZZLE CORNER



## MATHDOKU

Fill the grid with the numbers 1 to 8 such that each number appears only once in each row and column. The Grid is also divided in outlined regions called cages each with a given operator and target number. The numbers in the individual cells of a cage must produce that target number using the operator in a mathematical calculation.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 12+ | 3– | | 3– | | 3+ | | 48× |
| | 42× | 15× | | 2÷ | | 10+ | |
| | | 6× | | 4– | | | |
| 16+ | | | 5– | 7– | 2100× | | |
| 1– | 4÷ | | | | | | 21× |
| | 19+ | | | 144× | | | |
| 4÷ | 4 | | 48× | 5+ | | 80× | |
| | 6+ | | | | 1– | | |

*Readers can send their responses at editor@mtg.in or post us with complete address. Winners' names with their valuable feedback will be published in next issue.*

# beat the **TIME TRAP**

**Duration : 30 minutes**

## SECTION-I

### Single Option Correct Type

**1.** If $z_1$, $z_2$ are two complex numbers such that $\left|\frac{z_1 - z_2}{z_1 + z_2}\right| = 1$ and $iz_1 = kz_2$, where $k \in R$, then the angle between $z_1 - z_2$ and $z_1 + z_2$ is

(a) $\tan^{-1}\left(\frac{2k}{k^2+1}\right)$ (b) $\tan^{-1}\left(\frac{2k}{k^2-1}\right)$

(c) $-2\tan^{-1}k/2$ (d) $2\tan^{-1}k$

**2.** Let $5f(x) + 4f\left(\frac{1}{x}\right) = \frac{1}{x} + 3, x > 0$. Then $18\int_1^2 f(x)\,dx$ is equal to

(a) $5\log_e 2 + 3$ (b) $10\log_e 2 + 6$

(c) $5\log_e 2 - 3$ (d) $10\log_e 2 - 6$

**3.** A function $y = f(x)$ satisfies $f(x)\sin 2x + \sin x - (1 + \cos^2 x)f'(x) = 0$ with condition $f(0) = 0$. Then $f\left(\frac{\pi}{2}\right)$ is equal to

(a) 2 (b) 0 (c) 1 (d) $-1$

**4.** Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{c} = x\hat{i} + 2\hat{j} + 3\hat{k}, x \in R$. If $\vec{d}$ is the unit vector in the direction of $\vec{b} + \vec{c}$ such that $\vec{a} \cdot \vec{d} = 1$, then $(\vec{a} \times \vec{b}) \cdot \vec{c}$ is equal to

(a) 11 (b) 3 (c) 6 (d) 9

**5.** Let $f$ be a polynomial function such that $f(3x) = f'(x) \cdot f''(x)$, for all $x \in R$. Then

(a) $f(2) - f'(2) + f''(2) = 10$

(b) $f''(2) - f(2) = 4$

(c) $f''(2) - f'(2) = 0$

(d) $f(2) + f'(2) = 28$

**6.** If $P(6, 1)$ be the orthocentre of the triangle whose vertices are $A(5, -2)$, $B(8, 3)$ and $C(h, k)$, then the point $C$ lies on the circle

(a) $x^2 + y^2 - 65 = 0$ (b) $x^2 + y^2 - 74 = 0$

(c) $x^2 + y^2 - 52 = 0$ (d) $x^2 + y^2 - 61 = 0$

**7.** Let $S_k = \frac{1+2+\ldots+k}{k}$ and $\sum_{j=1}^{n} S_j^2 = \frac{n}{A}(Bn^2 + Cn + D)$, where $A, B, C, D \in N$ and $A$ has least value. Then

(a) $A + B$ is divisible by $D$

(b) $A + B + C + D$ is divisible by 5

(c) $A + C + D$ is not divisible by $B$

(d) $A + B = 5(D - C)$

**8.** Let $[x]$ denote the greatest integer $\leq x$, where $x \in R$. If the domain of the real valued function $f(x) = \sqrt{\frac{|[x]| - 2}{|[x]| - 3}}$ is $(-\infty, a) \cup [b, c) \cup [4, \infty)$, $a < b < c$, then the value of $a + b + c$ is

(a) 8 (b) $-2$ (c) $-3$ (d) 1

**9.** For $\alpha, \beta, \gamma \neq 0$, if $\sin^{-1}\alpha + \sin^{-1}\beta + \sin^{-1}\gamma = \pi$ and $(\alpha + \beta + \gamma)(\alpha - \gamma + \beta) = 3\alpha\beta$, then $\gamma$ equals

(a) $\frac{\sqrt{3}}{2}$ (b) $\sqrt{3}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{\sqrt{3}-1}{2\sqrt{2}}$

**10.** If $A$ is a square matrix of order 3 such that $\det(A) = 3$ and $\det(\text{adj}(-4\,\text{adj}(-3\,\text{adj}(3\,\text{adj}((2A)^{-1})))))) = 2^m 3^n$, then $m + 2n$ is equal to

(a) 6 (b) 2 (c) 3 (d) 4

## SECTION-II

### Numerical Answer Type

**11.** The number of 4-digited numbers of the form $abcd$ where $a, c, d < b$ and $a \neq 0$ is a four digited number then its tens place is _____.

**12.** Let $f: [-3, 1] \to R$ be given as

$$f(x) = \begin{cases} \min\{(x+6), x^2\}, & -3 \leq x \leq 0 \\ \max\{\sqrt{x}, x^2\}, & 0 \leq x \leq 1 \end{cases}.$$

If the area bounded by $y = f(x)$ and $x$-axis is $A$, then the value of $6A$ is equal to _____.

**13.** If $\lim\limits_{x\to 1}\dfrac{(5x+1)^{1/3}-(x+5)^{1/3}}{(2x+3)^{1/2}-(x+4)^{1/2}}=\dfrac{m\sqrt{5}}{n(2n)^{2/3}}$, where $gcd(m, n)= 1$, then $8m + 12n$ is equal to _____.

**14.** Let the sum of the coefficients of the first three terms in the expansion of $\left(x-\dfrac{3}{x^2}\right)^n$, $x \neq 0, n \in N$, be 376. Then the coefficient of $x^4$ is _____.

**15.** Let the set of all positive values of $\lambda$, for which the point of local minimum of the function $(1 + x(\lambda^2 - x^2))$ satisfies $\dfrac{x^2+x+2}{x^2+5x+6} < 0$ be $(\alpha, \beta)$. Then $\alpha^2 + \beta^2$ is equal to _____.

## SOLUTIONS

**1. (d) :** We have, $iz_1 = kz_2$ and $\left|\dfrac{z_1 - z_2}{z_1 + z_2}\right| = 1$

$$\Rightarrow \quad \frac{z_1 - \frac{iz_1}{k}}{z_1 + \frac{iz_1}{k}} = \cos\alpha + i\sin\alpha, \text{ where } \alpha \text{ is the angle between } z_1 - z_2 \text{ and } z_1 + z_2.$$

$$\Rightarrow \quad \frac{(k-i)^2}{k^2 - i^2} = \cos\alpha + i\sin\alpha$$

$$\Rightarrow \quad \cos\alpha = \frac{k^2-1}{k^2+1} \text{ and } \sin\alpha = \frac{-2k}{k^2+1}$$

$$\Rightarrow \quad \tan\alpha = \frac{-2k}{k^2-1} = \frac{2k}{1-k^2}$$

$$\Rightarrow \alpha = \tan^{-1}\left(\frac{2k}{1-k^2}\right) = 2\tan^{-1}k$$

**2. (d):** We have, $5f(x) + 4f\left(\dfrac{1}{x}\right) = \dfrac{1}{x} + 3$ ...(i)

Replace $x$ by $\dfrac{1}{x}$, we get $5f\left(\dfrac{1}{x}\right) + 4f(x) = x + 3$ ...(ii)

Solving (i) and (ii), we get

$$25f(x) - 16f(x) = \frac{5}{x} + 15 - 4x - 12$$

$$\Rightarrow \quad 9f(x) = \frac{5}{x} - 4x + 3 \Rightarrow \quad f(x) = \frac{1}{9}\left\{\frac{5}{x} - 4x + 3\right\}$$

$$\therefore \quad 18\int_1^2 f(x)\,dx = 18 \times \frac{1}{9}\int_1^2 \left(\frac{5}{x} - 4x + 3\right)dx$$

$$= 2\left[5\log_e x - \frac{4x^2}{2} + 3x\right]_1^2 = 2[5\log_e 2 - 6 + 3]$$

$$= 10\log_e 2 - 6$$

**3. (c) :** We have, $y\sin 2x + \sin x - (1 + \cos^2 x)\dfrac{dy}{dx} = 0$

$$\Rightarrow \quad \frac{dy}{dx}(1+\cos^2 x) - y\sin 2x = \sin x$$

$$\Rightarrow \quad \frac{dy}{dx} - \left(\frac{\sin 2x}{1+\cos^2 x}\right)y = \frac{\sin x}{1+\cos^2 x}$$

$$\therefore \quad \text{I.F.} = e^{-\int \frac{\sin 2x}{1+\cos^2 x}dx} = e^{\log\left(1+\cos^2 x\right)} = 1 + \cos^2 x$$

So, solution is given by

$$y \cdot (1+\cos^2 x) = \int \frac{\sin x}{(1+\cos^2 x)}(1+\cos^2 x)dx + c$$

$$\Rightarrow \quad y(1+\cos^2 x) = \int \sin x\,dx + c$$

$$\Rightarrow y(1+\cos^2 x) = -\cos x + c \qquad \text{...(i)}$$

Now, $y(0) = 0$

$$\Rightarrow \quad 0 = -\cos(0) + c \Rightarrow c = 1$$

From (i), $y(1 + \cos^2 x) = -\cos x + 1$

$$\Rightarrow \quad y = \frac{1-\cos x}{1+\cos^2 x}$$

$$\therefore \quad y\left(\frac{\pi}{2}\right) = \frac{1-0}{1+0} = 1$$

**4. (a) :** We have, $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$

and $\vec{c} = x\hat{i} + 2\hat{j} + 3\hat{k}$

Now, $\vec{b} + \vec{c} = (x + 2)\hat{i} + 6\hat{j} - 2\hat{k}$ and

$$\left|(\vec{b} + \vec{c})\right| = \sqrt{(x+2)^2 + (6)^2 + (-2)^2} = \sqrt{(x+2)^2 + 40}$$

Also, $\vec{d} = \dfrac{x+2}{\sqrt{40+(x+2)^2}}\hat{i} + \dfrac{6}{\sqrt{40+(x+2)^2}}\hat{j} - \dfrac{2}{\sqrt{40+(x+2)^2}}\hat{k}$

Given that $\vec{a} \cdot d = 1$

$$\Rightarrow \quad \frac{x+2+6-2}{\sqrt{40+(x+2)^2}} = 1$$

$$\Rightarrow \quad x + 6 = \sqrt{40 + (x+2)^2}$$

$$\Rightarrow \quad (x+6)^2 = 40 + (x+2)^2$$

$$\Rightarrow \quad x^2 + 36 + 12x = 40 + x^2 + 4 + 4x$$

$$\Rightarrow \quad x = 1$$

Now, $\left(\vec{a}\times\vec{b}\right)\cdot\vec{c}=\left[\vec{a}\ \vec{b}\ \vec{c}\right]=\begin{vmatrix}1 & 1 & 1\\ 2 & 4 & -5\\ 1 & 2 & 3\end{vmatrix}$

$= 1(12 + 10) - 1(6 + 5) + 1(4 - 4) = 11$

**5. (c) :** Let $f(x) = ax^3 + bx^2 + cx + d$

$\Rightarrow\ f(3x) = 27ax^3 + 9bx^2 + 3cx + d$

$f'(x) = 3ax^2 + 2bx + c, f''(x) = 6ax + 2b$

Now, $f(3x) = f'(x)\cdot f''(x)$

$\Rightarrow\ 27ax^3 + 9bx^2 + 3cx + d$

$= 18a^2x^3 + (6ab + 12ab)\,x^2 + (4b^2 + 6ac)x + 2bc$

Comparing coefficient of like powers, we get

$27a = 18a^2, 9b = 18ab, 3c = 4b^2 + 6ac, d = 2bc$

$\Rightarrow\ a = \frac{3}{2}, b = 0, c = 0, d = 0$

$\therefore\ f(x) = \frac{3}{2}x^3,\ f'(x) = \frac{9}{2}x^2,\ f''(x) = 9x$

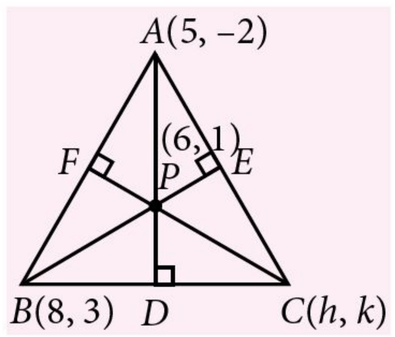
$\therefore\ f(2) = 12, f'(2) = 18, f''(2) = 18$

So, $f''(2) - f'(2) = 0$

**6. (a) :** Slope of $AP = \frac{1+2}{6-5} = 3$

Slope of $BC = \frac{-1}{\text{Slope of } AD}$

$= \frac{-1}{\text{Slope of } AP} = \frac{-1}{3}$




Equation of line $BC$ is given by

$(y - 3) = \frac{-1}{3}(x - 8)$

$\Rightarrow\ 3y + x - 17 = 0$ ...(i)

Now, slope of $BP = \frac{1-3}{6-8} = 1$

Slope of $AC = -1$

Equation of line $AC$ is given by $y + 2 = -1(x - 5)$

$\Rightarrow\ y + x - 3 = 0$ ...(ii)

Solving equation (i) and (ii), we get

$C = (-4, 7)$, which lies on circle $x^2 + y^2 - 65 = 0$.

**7. (a) :** We have, $S_k = \frac{1+2+...+k}{k}$

$\Rightarrow\ S_k = \frac{k(k+1)}{2k} = \frac{k+1}{2}$

Now, $\sum_{j=1}^{n}(S_j)^2 = \sum_{j=1}^{n}\left(\frac{j+1}{2}\right)^2 = \frac{1}{4}\sum_{j=1}^{n}(j+1)^2$

$= \frac{1}{4}[2^2 + 3^2 + 4^2 + ... + (n+1)^2]$

$= \frac{1}{4}[1^2 + 2^2 + 3^2 + 4^2 + ... + (n+1)^2 - 1^2]$

$= \frac{1}{4}\left[\frac{(n+1)(n+2)(2n+3)}{6} - 1\right]$

$= \frac{1}{4}\left(\frac{2n^3 + 9n^2 + 13n}{6}\right) = \frac{n}{24}(2n^2 + 9n + 13)$

$= \frac{n}{A}(Bn^2 + Cn + D)$ [Given]

$\therefore\ A = 24, B = 2, C = 9, D = 13$

So, $A + B = 24 + 2 = 26$

$\therefore\ A + B$ is divisible by $D$.

**8. (b):** For domain, $\frac{|[x]|-2}{|[x]|-3} \geq 0$

**Case I :** When $|[x]| - 2 \geq 0$ and $|[x]| - 3 > 0$

$\therefore\ x \in (-\infty, -3) \cup [4, \infty)$ ...(i)

**Case II :** When $|[x]| - 2 \leq 0$ and $|[x]| - 3 < 0$

$\therefore\ x \in [-2,3)$ ...(ii)

From (i) and (ii), we get

Domain of function $= (-\infty, -3) \cup [-2,3) \cup [4, \infty)$

$\therefore\ (a + b + c) = -3 + (-2) + 3 = -2$

**9. (a) :** Given, $\sin^{-1}\alpha + \sin^{-1}\beta + \sin^{-1}\gamma = \pi$

and $(\alpha + \beta + \gamma)(\alpha + \beta - \gamma) = 3\alpha\beta$

Let $\sin^{-1}\alpha = A$, $\sin^{-1}\beta = B$, $\sin^{-1}\gamma = C$

Now, $A + B + C = \pi$ and $(\alpha + \beta + \gamma)(\alpha + \beta - \gamma) = 3\alpha\beta$

$\Rightarrow\ \alpha^2 + \beta^2 + 2\alpha\beta - \gamma^2 = 3\alpha\beta \Rightarrow \alpha^2 + \beta^2 - \gamma^2 = \alpha\beta$

$\Rightarrow\ \frac{\alpha^2 + \beta^2 - \gamma^2}{2\alpha\beta} = \frac{1}{2}$

$\therefore\ \cos C = \frac{\alpha^2 + \beta^2 - \gamma^2}{2\alpha\beta} = \frac{1}{2} \Rightarrow C = 60°$

$\sin C = \frac{\sqrt{3}}{2} = \gamma$

So, $\gamma = \frac{\sqrt{3}}{2}$

**10. (d):** We have, $|A| = 3$

Now, $|\text{adj}\,(-4\ \text{adj}\,(-3\ \text{adj}\,(3\ \text{adj}\,(2A)^{-1})))|$

$= |-4\ \text{adj}\,(-3\ \text{adj}\,(3\ \text{adj}\,(2A)^{-1}))|^2$

[$\because\ |\text{adj}\,A| = |A|^{n-1}$, $n$ is order of $A$]

$= ((-4)^3)^2\,|\text{adj}\,(-3\ \text{adj}\,(3\ \text{adj}\,(2A)^{-1}))|^2$

$= 4^6\,|-3\ \text{adj}\,(3\ \text{adj}\,(2A)^{-1})|^4 = 4^6\,3^{12}\,|\text{adj}\,(3\ \text{adj}\,(2A)^{-1}|^4$

$= 4^6\,3^{12}\,|3\ \text{adj}\,(2A)^{-1}|^8 = 4^6\,3^{12}\,3^{24}\,|(2A)^{-1}|^{16}$

$= 4^6\,3^{36}\,2^{-48}\,|A^{-1}|^{16} = \frac{4^6\ 3^{36}}{2^{48}\cdot 3^{16}} = 2^{-36}\,3^{20}$

$\Rightarrow\ m = -36, n = 20$

$\therefore\ m + 2n = -36 + 40 = 4$

**11. (4):**

| $b$ | No. of ways |
|---|---|
| 2 | $1 \cdot 2^2$ |
| 3 | $2 \cdot 3^2$ |
| 4 | $3 \cdot 4^2$ |
| . | . |
| . | . |
| 9 | $8 \cdot 9^2$ |

Required number of ways

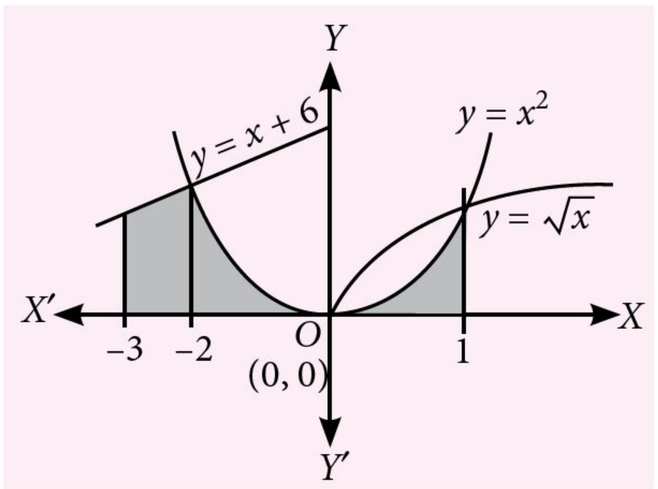
$$= \sum_{n=1}^{8} n(n+1)^2 = \sum_{n=1}^{8} (n+1)^3 - \sum_{n=1}^{8} 1(n+1)^2$$

$$= (2^3 + 3^3 + .... + 9^3) - (2^2 + 3^2 + .... + 9^2)$$

$$= (45)^2 - \frac{9 \times 10 \times 19}{6} = 1740$$

$\therefore$ Tens place = 4

**12. (41):**




Area bounded by $y = f(x)$ and $x$-axis,

$$A = \int_{-3}^{-2} (x+6)\,dx + \int_{-2}^{0} x^2\,dx + \int_{0}^{1} \sqrt{x}\,dx$$

$$= \left[\frac{x^2}{2} + 6x\right]_{-3}^{-2} + \left[\frac{x^3}{3}\right]_{-2}^{0} + \left[\frac{x^{3/2}}{3/2}\right]_{0}^{1} = \frac{7}{2} + \frac{8}{3} + \frac{2}{3}$$

$\therefore \quad A = \frac{41}{6} \Rightarrow 6A = 41$

**13. (100):** Let $L = \lim\limits_{x \to 1} \dfrac{(5x+1)^{1/3} - (x+5)^{1/3}}{(2x+3)^{1/2} - (x+4)^{1/2}} \left(\dfrac{0}{0} \text{ form}\right)$

Using L-Hospital's rule

$$L = \lim_{x \to 1} \frac{\frac{1}{3} \times 5(5x+1)^{-2/3} - \frac{1}{3}(x+5)^{-2/3}}{\frac{1}{2} \times 2(2x+3)^{-1/2} - \frac{1}{2}(x+4)^{1/2}}$$

$$= \frac{\left(\frac{5}{3} - \frac{1}{3}\right)6^{-2/3}}{\left(\frac{1}{2}\right)5^{-1/2}} = \frac{8}{3} \times \frac{5^{1/2}}{6^{2/3}} = \frac{m\sqrt{5}}{n(2n)^{2/3}}$$

$\Rightarrow \quad m = 8, n = 3$

$\Rightarrow \quad 8m + 12n = 100$

**14. (405) :** Sum of coefficient of first three terms of $\left(x - \dfrac{3}{x^2}\right)^n$ is 376.

$\because \quad {}^nC_0 - {}^nC_1 \cdot 3 + {}^nC_2 \cdot 3^2 = 376$

$\Rightarrow \quad 1 - 3n + \dfrac{n(n-1)}{2} \times 9 = 376$

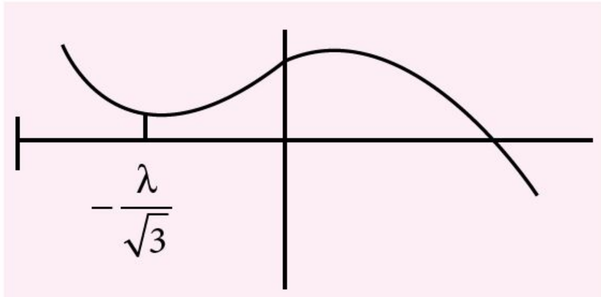
$\Rightarrow \quad 3n^2 - 5n - 250 = 0 \Rightarrow n = 10$

Now, $T_{r+1} = {}^{10}C_r\, x^{10-r} \cdot \left(\dfrac{-3}{x^2}\right)^r = {}^{10}C_r\,(-3)^r \cdot x^{10-3r}$

For coefficient of $x^4$, $10 - 3r = 4 \Rightarrow r = 2$

$\therefore \quad$ Coefficient of $x^4 = {}^{10}C_2\,(-3)^2 = 405$

**15. (39) :**




Let $f(x) = 1 + x(\lambda^2 - x^2)$

$\Rightarrow \quad f(x) = -x^3 + \lambda^2 x + 1$

$\Rightarrow \quad f'(x) = -3x^2 + \lambda^2$

For maxima or minima, $f'(x) = 0$

$\Rightarrow \quad \lambda^2 - 3x^2 = 0$

$\Rightarrow \quad (\lambda - \sqrt{3}x)(\lambda + \sqrt{3}x) = 0$

$\Rightarrow \quad x = \pm\dfrac{\lambda}{\sqrt{3}}$

Now, $f''(x) = -6x$,

So, $x = \dfrac{-\lambda}{\sqrt{3}}$ is point of minima.

Now, $-\dfrac{\lambda}{\sqrt{3}}$ should satisfy the given condition

$$\frac{x^2 + x + 2}{x^2 + 5x + 6} < 0$$

$\Rightarrow \quad \dfrac{1}{(x+2)(x+3)} < 0 \qquad [\because \ x^2 + x + 2 > 0]$

$\Rightarrow \quad x \in (-3, -2)$

$\Rightarrow \quad -3 < -\dfrac{\lambda}{\sqrt{3}} < -2$

$\Rightarrow -3\sqrt{3} < -\lambda < -2\sqrt{3}$

$\Rightarrow \quad 2\sqrt{3} < \lambda < 3\sqrt{3}$

$\Rightarrow \lambda \in (2\sqrt{3}, 3\sqrt{3}) \equiv (\alpha, \beta) \qquad (\because \text{ Given})$

$\therefore \quad \alpha^2 + \beta^2 = \left(2\sqrt{3}\right)^2 + \left(3\sqrt{3}\right)^2 = 12 + 27 = 39$

# Quiz CLUB

Unlock Your Knowledge!

1. Find the points on $y$-axis whose perpendicular distance from the line $4x - 3y - 12 = 0$ is 3 units.
2. Find the domain of the continuity of the function $f(x) = \sin^{-1}x - [x]$, where $[x]$ denotes the greatest integer less than or equal to $x$.
3. Let $A = \{x \in R : -4 \le x \le 4\}$ and $x \ne 0$. If $f: A \to R$ is defined by $f(x) = \frac{|x|}{x}$, then find the range of $f$.
4. If $\sec^2\theta + \text{cosec}^2\theta$ is always greater than or equal to $k$, then find $k$.
5. If $A$ and $B$ are two matrices of the order $3 \times m$ and $3 \times n$, respectively and $m = n$, then find the order of matrix $5A - 2B$.
6. Evaluate : $\lim\limits_{x\to 0} \frac{e^x - e^{\sin x}}{x - \sin x}$
7. Find the value of $\int \frac{1}{x(x^n + 1)} dx$.
8. If $g(x) = x^2 - 10x - 18$. Determine the image of $-2$ and the pre-image of 6 respectively.
9. Mean and standard deviation of 100 items are 50 and 4, respectively. Find the sum of the squares of items.
10. Evaluate : $\int_{-\pi/2}^{\pi/2} 2\sin|x| + \cos|x|\, dx$ .
11. Find the value of $(32)\cdot(32)^{1/6}\cdot(32)^{1/36}\cdot\ldots\infty$.
12. Which is greater, $\tan 1$ or $\tan^{-1} 1$?
13. If $\left(\frac{1-i}{1+i}\right)^n = 1$, then find the least positive integral value of $n$.
14. If $A$ is a square matrix of order 3 such that $|\text{adj } A| = 64$, then find the value of $|A|$.
15. If $(-3, 0)$ is the vertex and $y$-axis is the directrix of a parabola, then find its focus.
16. Find the interval in which the function $f(x) = x - e^x + \tan(2\pi/7)$ is increases.
17. Find the L.C.M. of 5!, 6! and 7!.
18. Draw the graph of $y = |x + 1|$ and using integration, find the area below $y = |x + 1|$, above $X$ - axis and between $x = -4$ to $x = 2$.
19. Solve the following differential equation $(1 + y^2)\tan^{-1}x\,dx + 2y(1 + x^2)dy = 0$.
20. Which is larger $(1.01)^{100000}$ or 1000?

*Readers can send their responses at editor@mtg.in or post us with complete address by 10th of every month. Winners' names and answers will be published in next issue.*

## SOLUTIONS TO JUNE 2025 QUIZ CLUB

1. $\{3, 5, 11, 13\}$
2. 0.5
3. 2
4. $x = 100$
5. 2019!
6. 1
7. $[d - a, d + a]$
8. 3
9. $\alpha \in (-\infty, -2] \cup [8, \infty)$
10. $2\sqrt{7}$
11. 0
12. 113
13. 1 : 3
14. $= \frac{9\left(\frac{1}{5}\right)^x}{\log\left(\frac{1}{5}\right)} - 25\frac{\left(\frac{1}{3}\right)^x}{\log\left(\frac{1}{3}\right)} + c$
15. 9
16. $5^{10}$
17. 25
18. $y^2 - 10x - 6y + 14 = 0$
19. $\alpha = 1$
20. ${}^{20}C_{10}$

# Maths in FOCUS

**TOPIC**

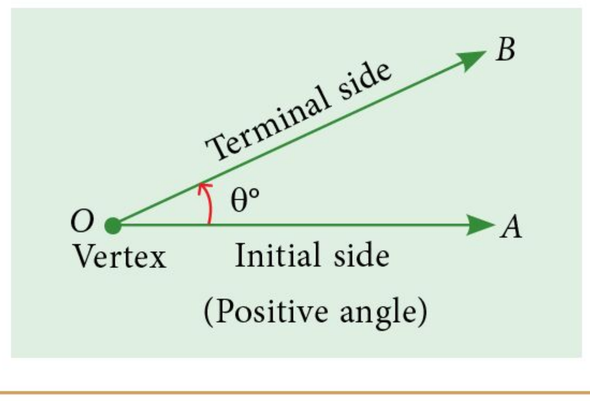
**Trigonometric and Inverse Trigonometric Functions**

*Detailed theory with High Definition images of the given topic is covered under this heading.*

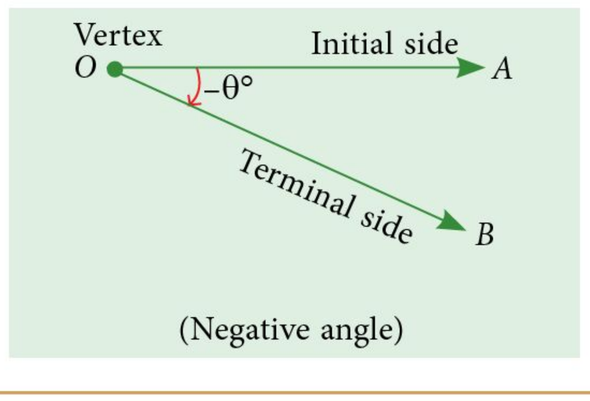
# Trigonometric Functions

## Measure of an Angle

Angles is the amount of rotation of a given ray from the initial side to its terminal side about its vertex.




(Positive angle)

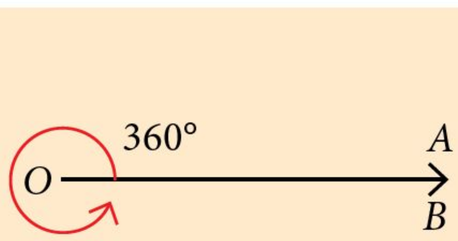


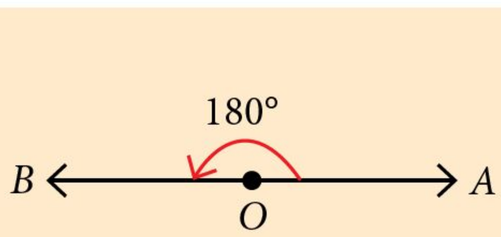

(Negative angle)

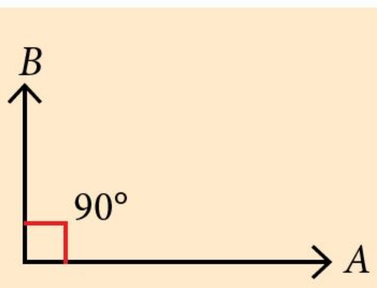
## Degree Measure

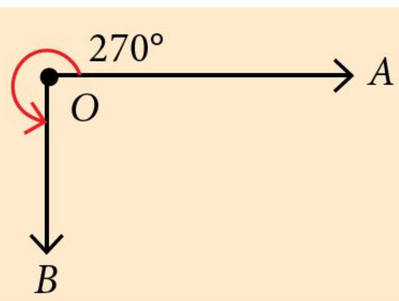
The angle is said to have a measure of one degree, if a rotation from the initial side to terminal side is $1/360^{th}$ of a revolution. It is represented as 1°.

Note :1° (1 degree) = 60′ (60 minutes), Also, 1′ (1 minute) = 60″ (60 seconds).



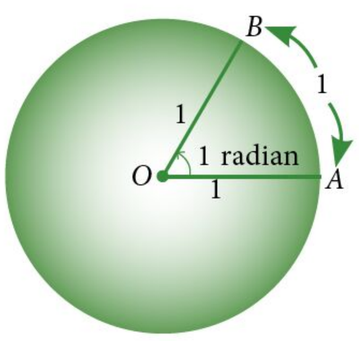



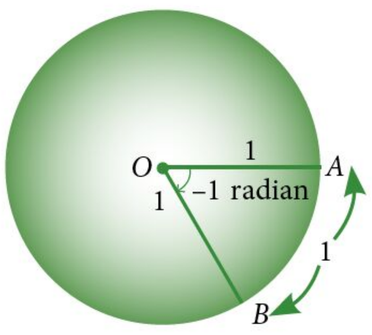




## Radian Measure

The angle is said to have a measure of 1 radian if it is the angle measures at the centre by an arc of length 1 unit in a circle.




## Relation between Degree and Radian Measure

$$\text{Radian Measure} = \frac{\pi}{180} \times \text{Degree Measure}$$

$$\text{Degree Measure} = \frac{180}{\pi} \times \text{Radian Measure}$$

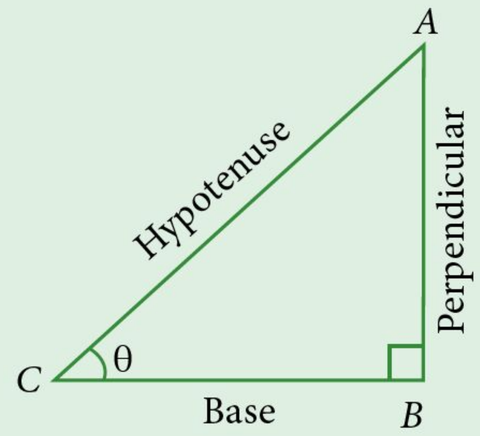
## Trigonometric Ratios for Acute Angle

The relation between different sides and angles of a right angled triangle are called trigonometric ratios.

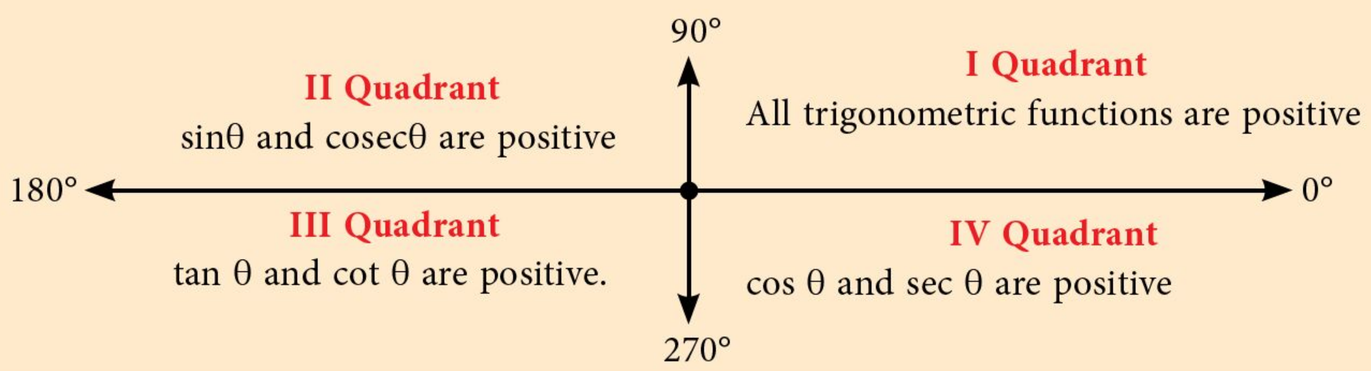
$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$; $\quad \text{cosec}\,\theta = \frac{\text{Hypotenuse}}{\text{Perpendicular}}$;

$\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}}$; $\quad \sec\theta = \frac{\text{Hypotenuse}}{\text{Base}}$;

$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$; $\quad \cot\theta = \frac{\text{Base}}{\text{Perpendicular}}$



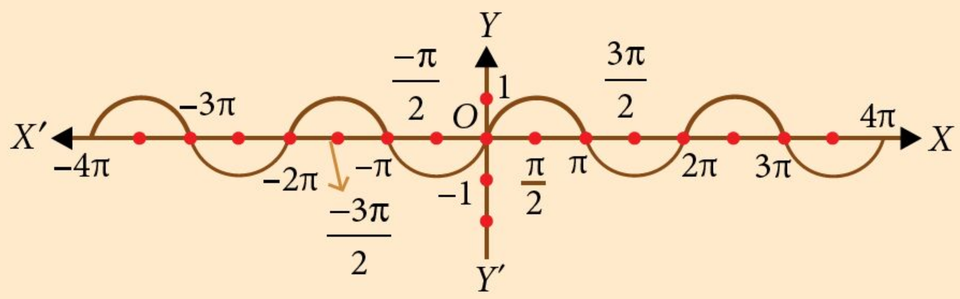

## Sign of Trigonometric Functions



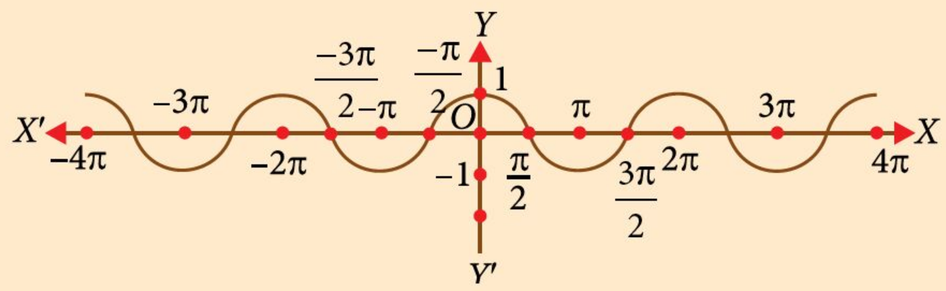

## Domain and Range of Trigonometric Functions

| Function | Domain | Range |
|---|---|---|
| $y = \sin x$ | Set of all real numbers | $[-1, 1]$ ( $\because -1 \le y \le 1$) |
| $y = \cos x$ | Set of all real numbers | $[-1, 1]$ ( $\because -1 \le y \le 1$) |
| $y = \text{cosec}\,x$ | $\{x : x \in R$ and $x \ne n\pi, n \in Z\}$ | $\{y : y \in R, y \ge 1$ or $y \le -1\}$ |
| $y = \sec x$ | $\{x : x \in R$ and $x \ne (2n+1)\frac{\pi}{2}, n \in Z\}$ | $\{y : y \in R, y \le -1$ or $y \ge 1\}$ |
| $y = \tan x$ | $\{x : x \in R$ and $x \ne (2n+1)\frac{\pi}{2}, n \in Z\}$ | Set of all real numbers |
| $y = \cot x$ | $\{x : x \in R$ and $x \ne n\pi, n \in Z\}$ | Set of all real numbers |

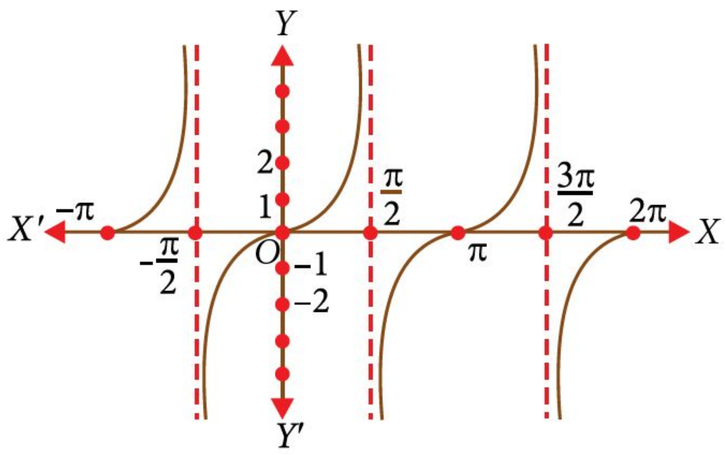
## Graphs of Trigonometric Functions



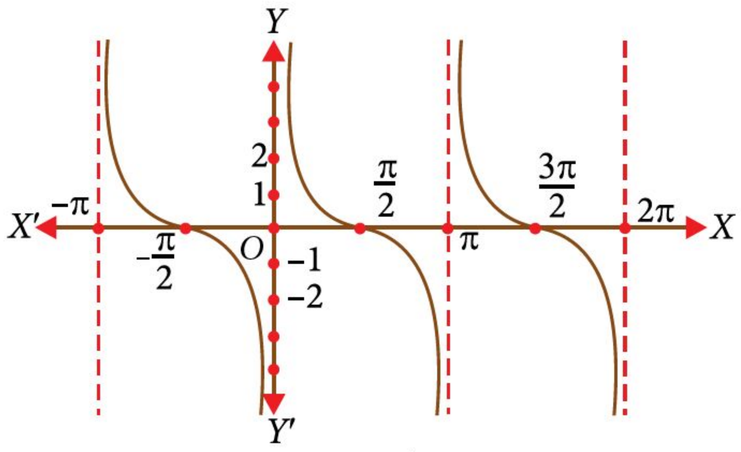

$y = \sin x$



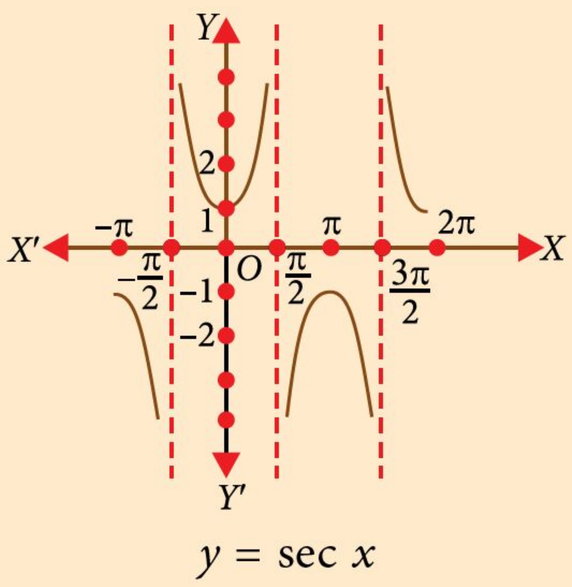

$y = \cos x$



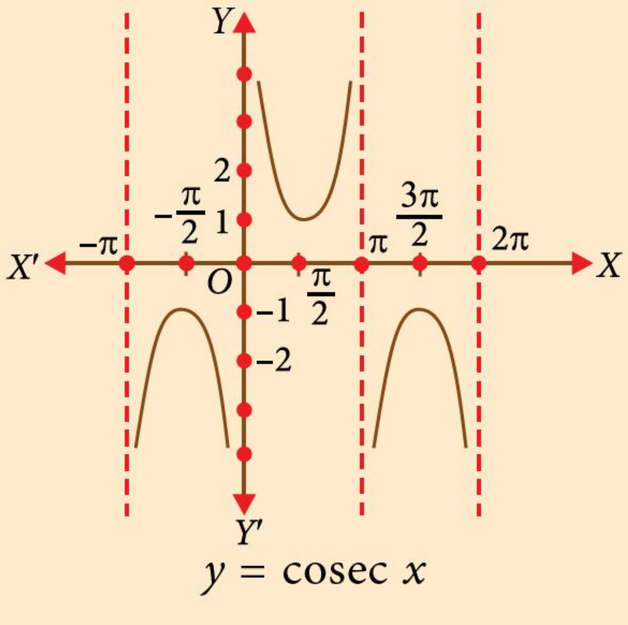

$y = \tan x$




$y = \cot x$




$y = \sec x$




$y = \text{cosec } x$

## Trigonometric Functions in Terms of Sine and Cosine Functions

- $\text{cosec } \theta = \dfrac{1}{\sin\theta}$, $\theta \neq n\pi$, where $n$ is any integer.
- $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$, $\theta \neq (2n+1)\dfrac{\pi}{2}$, where $n$ is any integer.
- $\sec\theta = \dfrac{1}{\cos\theta}$, $\theta \neq (2n+1)\dfrac{\pi}{2}$, where $n$ is any integer.
- $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$, $\theta \neq n\pi$, where $n$ is any integer.

# CONCEPT MAP

# TRIGONOMETRIC FUNCTIONS

CLASS XI CLASS XII

## Relationship between Degree and Radian

- 1 radian = $\frac{180^\circ}{\pi}$ = 57°16′ approximately
- $1^\circ = \left(\frac{\pi}{180}\right)$ radian = 0.01746 radian approximately

## Sum and Difference of Two Angles

- $\sin(\theta \pm \phi) = \sin\theta\cos\phi \pm \cos\theta\sin\phi$
- $\cos(\theta \pm \phi) = \cos\theta\cos\phi \mp \sin\theta\sin\phi$
- $\tan(\theta \pm \phi) = \frac{\tan\theta \pm \tan\phi}{1 \mp \tan\theta \cdot \tan\phi}$
- $\cot(\theta \pm \phi) = \frac{\cot\theta \cdot \cot\phi \mp 1}{\cot\phi \pm \cot\theta}$
- $\sin\theta + \sin\phi = 2\sin\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right)$
- $\sin\theta - \sin\phi = 2\cos\left(\frac{\theta+\phi}{2}\right)\sin\left(\frac{\theta-\phi}{2}\right)$
- $\cos\theta + \cos\phi = 2\cos\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right)$
- $\cos\theta - \cos\phi = -2\sin\left(\frac{\theta+\phi}{2}\right)\sin\left(\frac{\theta-\phi}{2}\right)$
- $2\sin\theta\cos\phi = \sin(\theta+\phi) + \sin(\theta-\phi)$
- $2\cos\theta\sin\phi = \sin(\theta+\phi) - \sin(\theta-\phi)$
- $2\cos\theta\cos\phi = \cos(\theta+\phi) + \cos(\theta-\phi)$
- $2\sin\theta\sin\phi = \cos(\theta-\phi) - \cos(\theta+\phi)$

## Trigonometric Function of Some Standard Angles

| Angle | 0° | 30° | 45° | 60° | 90° | 120° | 135° | 150° | 180° |
|---|---|---|---|---|---|---|---|---|---|
| sin | 0 | $\frac{1}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{\sqrt{3}}{2}$ | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{2}$ | 0 |
| cos | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{2}$ | 0 | $-\frac{1}{2}$ | $-\frac{1}{\sqrt{2}}$ | $-\frac{\sqrt{3}}{2}$ | -1 |
| tan | 0 | $\frac{1}{\sqrt{3}}$ | 1 | $\sqrt{3}$ | ∞ | $-\sqrt{3}$ | -1 | $-\frac{1}{\sqrt{3}}$ | 0 |

| Angle | 0° | 30° | 45° | 60° | 90° | 120° | 135° | 150° | 180° |
|---|---|---|---|---|---|---|---|---|---|
| cot | ∞ | $\sqrt{3}$ | 1 | $\frac{1}{\sqrt{3}}$ | 0 | $-\frac{1}{\sqrt{3}}$ | -1 | $-\sqrt{3}$ | -∞ |
| sec | 1 | $\frac{2}{\sqrt{3}}$ | $\sqrt{2}$ | 2 | ∞ | -2 | $-\sqrt{2}$ | $-\frac{2}{\sqrt{3}}$ | -1 |
| cosec | ∞ | 2 | $\sqrt{2}$ | $\frac{2}{\sqrt{3}}$ | 1 | $\frac{2}{\sqrt{3}}$ | $\sqrt{2}$ | 2 | ∞ |

## Functions Domain Range

| | | |
|---|---|---|
| $\sin\theta$ | $R$ | $[-1, 1]$ |
| $\cos\theta$ | $R$ | $[-1, 1]$ |
| $\tan\theta$ | $R - \left\{(2n+1)\frac{\pi}{2}, n \in Z\right\}$ | $R$ |
| $\sec\theta$ | $R - \left\{(2n+1)\frac{\pi}{2}, n \in Z\right\}$ | $R - (-1, 1)$ |
| $\operatorname{cosec}\theta$ | $R - \{n\pi, n \in Z\}$ | $R - (-1, 1)$ |
| $\cot\theta$ | $R - \{n\pi, n \in Z\}$ | $R$ |

## Trigonometric Identities

- $\sin^2\theta + \cos^2\theta = 1$
- $1 + \tan^2\theta = \sec^2\theta$
- $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

## Multiple Angles

- $\sin 2\theta = 2\sin\theta\cos\theta = \frac{2\tan\theta}{1+\tan^2\theta}$
- $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$
  $= 1 - 2\sin^2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$
- $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$
- $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$
- $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$
- $\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$

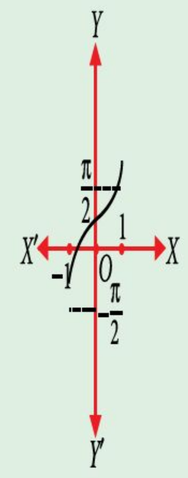
# INVERSE TRIGONOMETRIC FUNCTIONS

# CONCEPT MAP

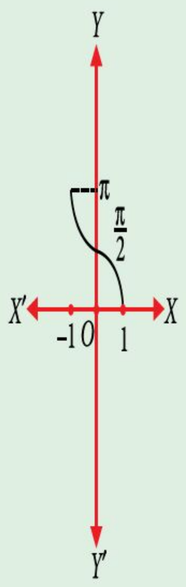
| Function | Domain | Range |
|---|---|---|
| $\sin^{-1}\theta$ | $[-1, 1]$ | $[-\pi/2, \pi/2]$ |
| $\cos^{-1}\theta$ | $[-1, 1]$ | $[0, \pi]$ |
| $\tan^{-1}\theta$ | $R$ | $(-\pi/2, \pi/2)$ |
| $\cot^{-1}\theta$ | $R$ | $(0, \pi)$ |
| $\operatorname{cosec}^{-1}\theta$ | $R - (-1, 1)$ | $[-\pi/2, \pi/2] - \{0\}$ |
| $\sec^{-1}\theta$ | $R - (-1, 1)$ | $[0, \pi] - \{\pi/2\}$ |

## Graphs of Inverse Trigonometric Functions

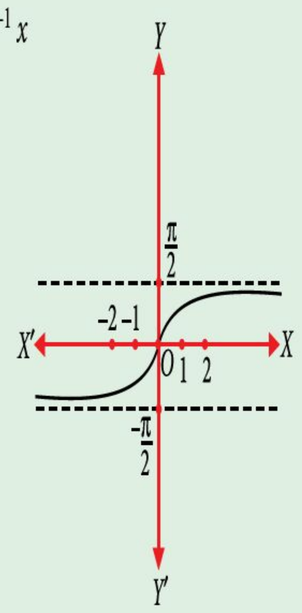
$y = \sin^{-1} x$



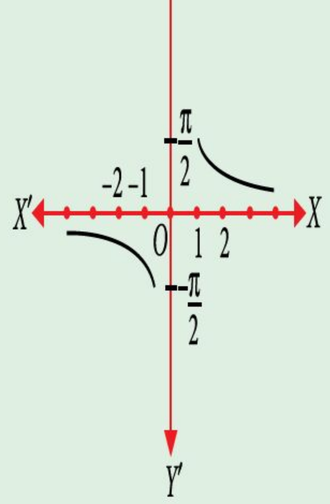

$y = \cos^{-1} x$



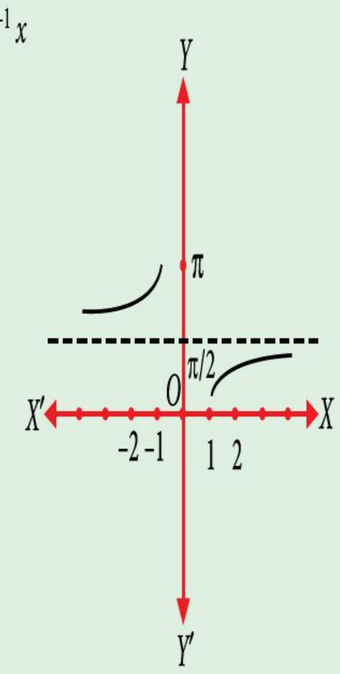

$y = \tan^{-1} x$



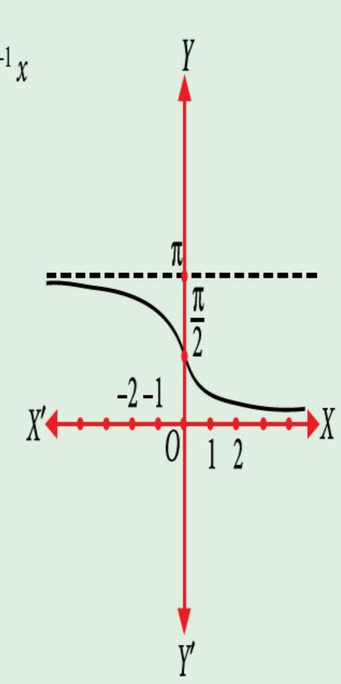

$y = \operatorname{cosec}^{-1} x$




$y = \sec^{-1} x$




$y = \cot^{-1} x$



## Fundamental Trigonometric Identities

The relation involving trigonometric functions, which are true for all values of the angles involved are called trigonometric identities.

- $\sin^2\theta + \cos^2\theta = 1$
- $1 + \tan^2\theta = \sec^2\theta$
- $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

**Transformation of Trigonometric Ratios in Terms of other Trigonometric Ratios**

| | $\sin\theta$ | $\cos\theta$ | $\tan\theta$ | $\cot\theta$ | $\sec\theta$ | $\operatorname{cosec}\theta$ |
|---|---|---|---|---|---|---|
| $\sin\theta$ | $\sin\theta$ | $\pm\sqrt{1-\cos^2\theta}$ | $\pm\frac{\tan\theta}{\sqrt{1+\tan^2\theta}}$ | $\pm\frac{1}{\sqrt{1+\cot^2\theta}}$ | $\pm\frac{\sqrt{\sec^2\theta-1}}{\sec\theta}$ | $\frac{1}{\operatorname{cosec}\theta}$ |
| $\cos\theta$ | $\pm\sqrt{1-\sin^2\theta}$ | $\cos\theta$ | $\pm\frac{1}{\sqrt{1+\tan^2\theta}}$ | $\pm\frac{\cot\theta}{\sqrt{1+\cot^2\theta}}$ | $\frac{1}{\sec\theta}$ | $\pm\frac{\sqrt{\operatorname{cosec}^2\theta-1}}{\operatorname{cosec}\theta}$ |
| $\tan\theta$ | $\pm\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$ | $\pm\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$ | $\tan\theta$ | $\frac{1}{\cot\theta}$ | $\pm\sqrt{\sec^2\theta-1}$ | $\pm\frac{1}{\sqrt{\operatorname{cosec}^2\theta-1}}$ |
| $\cot\theta$ | $\pm\frac{\sqrt{1-\sin^2\theta}}{\sin\theta}$ | $\pm\frac{\cos\theta}{\sqrt{1-\cos^2\theta}}$ | $\frac{1}{\tan\theta}$ | $\cot\theta$ | $\pm\frac{1}{\sqrt{\sec^2\theta-1}}$ | $\pm\sqrt{\operatorname{cosec}^2\theta-1}$ |
| $\sec\theta$ | $\pm\frac{1}{\sqrt{1-\sin^2\theta}}$ | $\frac{1}{\cos\theta}$ | $\pm\sqrt{1+\tan^2\theta}$ | $\pm\frac{\sqrt{1+\cot^2\theta}}{\cot\theta}$ | $\sec\theta$ | $\pm\frac{\operatorname{cosec}\theta}{\sqrt{\operatorname{cosec}^2\theta-1}}$ |
| $\operatorname{cosec}\theta$ | $\frac{1}{\sin\theta}$ | $\pm\frac{1}{\sqrt{1-\cos^2\theta}}$ | $\pm\frac{\sqrt{1+\tan^2\theta}}{\tan\theta}$ | $\pm\sqrt{1+\cot^2\theta}$ | $\pm\frac{\sec\theta}{\sqrt{\sec^2\theta-1}}$ | $\operatorname{cosec}\theta$ |

## Trigonometric Functions of Compound Angles

- $\sin(\theta+\phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$
- $\sin(\theta-\phi) = \sin\theta\cos\phi - \cos\theta\sin\phi$
- $\cos(\theta+\phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$
- $\cos(\theta-\phi) = \cos\theta\cos\phi + \sin\theta\sin\phi$
- $\tan(\theta+\phi) = \dfrac{\tan\theta+\tan\phi}{1-\tan\theta\tan\phi}$
- $\tan(\theta-\phi) = \dfrac{\tan\theta-\tan\phi}{1+\tan\theta\tan\phi}$
- $\cot(\theta+\phi) = \dfrac{\cot\theta\cot\phi-1}{\cot\phi+\cot\theta}$
- $\cot(\theta-\phi) = \dfrac{\cot\theta\cot\phi+1}{\cot\phi-\cot\theta}$

## Values of T-Functions for Some Particular Angles

The values of trigonometric functions for these angles are same as that of trigonometric ratios and are given in the following table.

| T-Functions / Angles | sin | cos | tan | cot | sec | cosec |
|---|---|---|---|---|---|---|
| $\frac{\pi}{12}$ | $\frac{\sqrt{3}-1}{2\sqrt{2}}$ | $\frac{\sqrt{3}+1}{2\sqrt{2}}$ | $2-\sqrt{3}$ | $2+\sqrt{3}$ | $\sqrt{2}(\sqrt{3}-1)$ | $\sqrt{2}(\sqrt{3}+1)$ |
| $\frac{\pi}{10}$ | $\frac{\sqrt{5}-1}{4}$ | $\frac{\sqrt{10+2\sqrt{5}}}{4}$ | $\frac{1}{5}\sqrt{25-10\sqrt{5}}$ | $\sqrt{5+2\sqrt{5}}$ | $\frac{1}{5}\sqrt{50-10\sqrt{5}}$ | $1+\sqrt{5}$ |
| $\frac{\pi}{6}$ | $\frac{1}{2}$ | $\frac{\sqrt{3}}{2}$ | $\frac{1}{\sqrt{3}}$ | $\sqrt{3}$ | $\frac{2}{\sqrt{3}}$ | 2 |
| $\frac{\pi}{5}$ | $\frac{\sqrt{10-2\sqrt{5}}}{4}$ | $\frac{\sqrt{5}+1}{4}$ | $\sqrt{5-2\sqrt{5}}$ | $\frac{\sqrt{25+10\sqrt{5}}}{5}$ | $\sqrt{5}-1$ | $\frac{1}{5}\sqrt{50+10\sqrt{5}}$ |
| $\frac{\pi}{4}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$ | 1 | 1 | $\sqrt{2}$ | $\sqrt{2}$ |
| $\frac{\pi}{3}$ | $\frac{\sqrt{3}}{2}$ | $\frac{1}{2}$ | $\sqrt{3}$ | $\frac{1}{\sqrt{3}}$ | 2 | $\frac{2}{\sqrt{3}}$ |
| $\pi/2$ | 1 | 0 | not defined | 0 | not defined | 1 |
| $\pi$ | 0 | $-1$ | 0 | not defined | $-1$ | not defined |
| $3\pi/2$ | $-1$ | 0 | not defined | 0 | not defined | $-1$ |
| $2\pi$ | 0 | 1 | 0 | not defined | 1 | not defined |

## Transformation Formulae

- $2\sin\theta\cos\phi = \sin(\theta+\phi)+\sin(\theta-\phi)$
- $2\cos\theta\sin\phi = \sin(\theta+\phi)-\sin(\theta-\phi)$
- $2\cos\theta\cos\phi = \cos(\theta+\phi)+\cos(\theta-\phi)$
- $2\sin\theta\sin\phi = \cos(\theta-\phi)-\cos(\theta+\phi)$
- $\sin\theta+\sin\phi = 2\sin\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right)$
- $\sin\theta-\sin\phi = 2\cos\left(\frac{\theta+\phi}{2}\right)\sin\left(\frac{\theta-\phi}{2}\right)$
- $\cos\theta+\cos\phi = 2\cos\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right)$
- $\cos\theta-\cos\phi = -2\sin\left(\frac{\theta+\phi}{2}\right)\sin\left(\frac{\theta-\phi}{2}\right)$
  $= 2\sin\left(\frac{\theta+\phi}{2}\right)\sin\left(\frac{\phi-\theta}{2}\right)$

## Trigonometric Functions of Multiple Angles

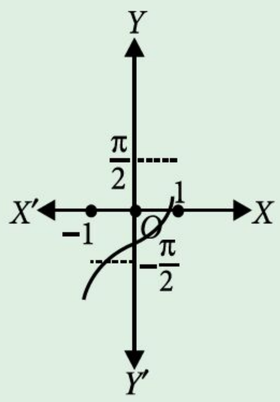
- $\sin 2\theta = 2\sin\theta\cos\theta = \dfrac{2\tan\theta}{1+\tan^2\theta}$
- $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$

$$= 1 - 2\sin^2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$$

- $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$
- $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$
- $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$
- $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$

# Inverse Trigonometric Functions

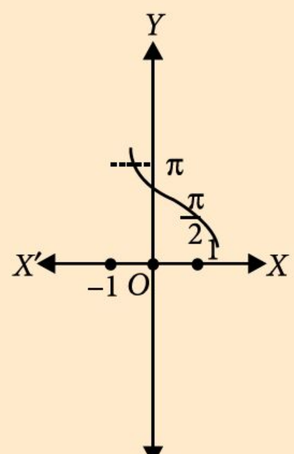
## Graphs of Inverse Trigonometric Functions




$y = \sin^{-1}x$

**Domain :** $[-1, 1]$

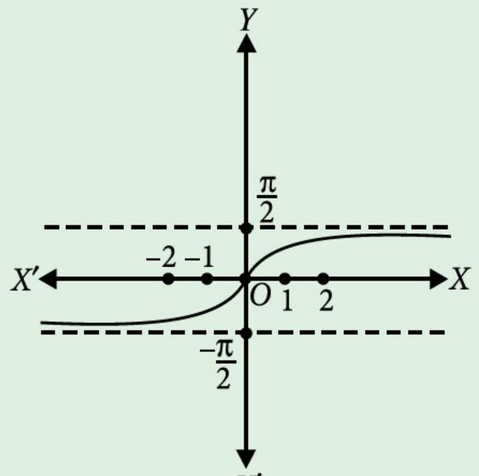
**Range :** $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$




$y = \cos^{-1}x$

**Domain :** $[-1, 1]$

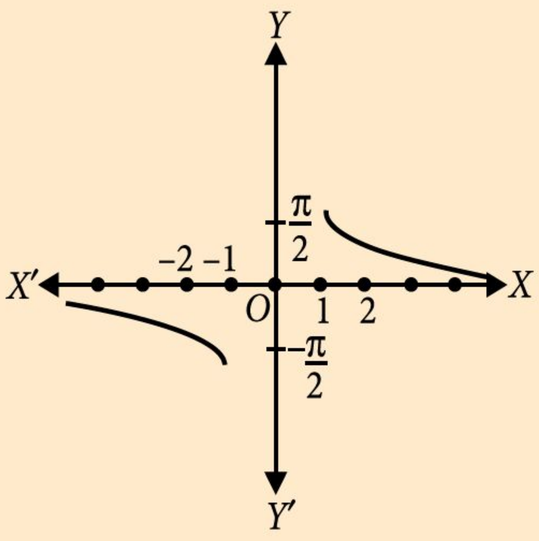
**Range :** $[0, \pi]$




$y = \tan^{-1}x$

**Domain :** $R$

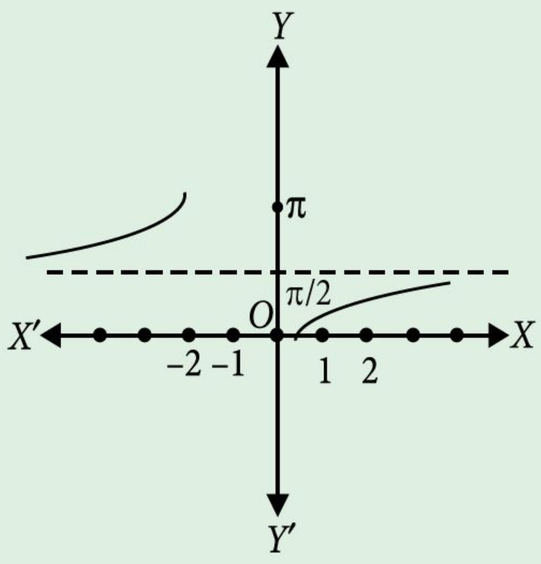
**Range :** $\left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$




$y = \text{cosec}^{-1}x$

**Domain :** $R - ]-1, 1[$

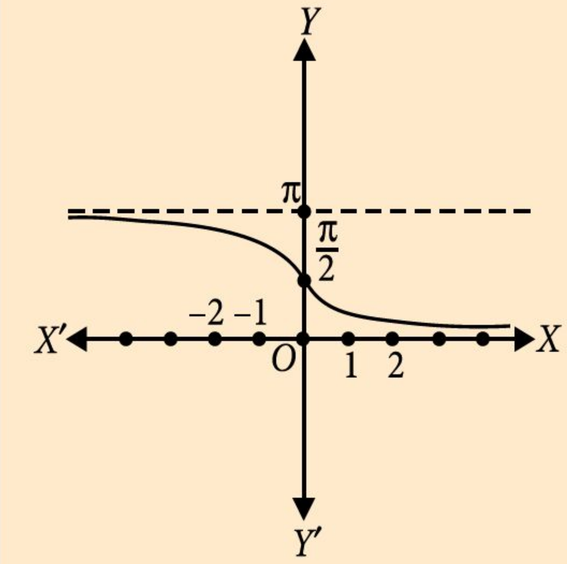
**Range :** $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$




$y = \sec^{-1}x$

**Domain :** $R - ]-1, 1[$

**Range :** $[0,\pi] - \left\{\frac{\pi}{2}\right\}$




$y = \cot^{-1}x$

**Domain :** $R$

**Range :** $]0, \pi[$

# Brain Teaser

# LOGICAL REASONING

## For Various Competitive Exams

**1.** Find the next term in given series.

YXHGWVFEUTDCSRB

(a) D (b) C (c) G (d) A

***Direction Q.(2 and 3) : In each of the following questions, identify what will come in the place of the blank?***

**2.** JKLM : lmjk:: TUVW : .....

(a) utwr (b) vwtu

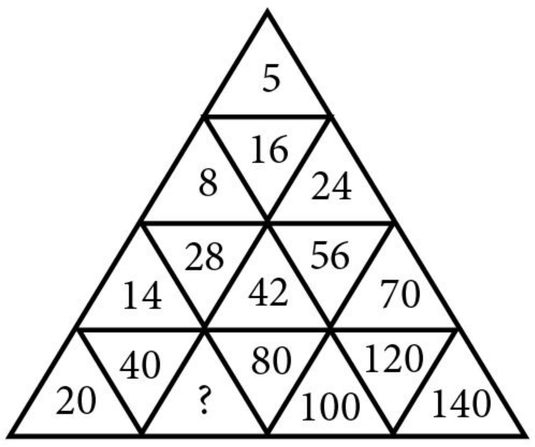
(c) vtwu (d) vtuw

**3.** 839 : 919 :: 1012 : .....

(a) 1052 (b) 1072

(c) 1132 (d) 1093

**4.** Find the number/letter which replaces the sign of '?'




(a) 50 (b) 60 (c) 70 (d) 55

***Direction Q.(5 and 6) : Read the following information to answer the questions:***

| Number | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| Symbol | × | * | ? | ÷ | $ | • | + | ! | D |

Following conditions are to be observed:

**I.** If any number begins by odd number, then that odd number symbolised by @.

**II.** If any number ends by even number, then that even number symbolised as ©.

**5.** Which of the following will be the symbol of 673258?

(a) •+?*$! (b) @+?*$!

(c) @?+$*! (d) •+?*$©

**6.** Which of the following will be the symbol of 236475?

(a) ?*•÷×@

(b) ©?•÷+@

(c) ×?•÷+@

(d) None of these

**7.** A cube has six faces each of a different colour. The red face is opposite to black. The green face is in between red and black. The blue face is adjacent to white and brown face is adjacent to blue. The four colours adjacent to green are

(a) red, black, brown and white

(b) red, black, brown and blue

(c) red, black, blue and white

(d) red, brown, blue and white

**8.** If '8 × C' means '8 is the daughter of C', '8 + C' means '8 is the husband of C' and '8 – C' means '8 is the sister of C', then what does 'M + N – P × Q' mean?

(a) M is the brother-in-law of Q

(b) M is the uncle of Q

(c) M is the son-in-law of Q

(d) Q is the mother of M

***Direction Q.(9 and 10) : Read the following information carefully and answer the questions that follow:***

Point H is 6 m towards the East of point G, point R is 8 m North of point G. Point Q is exactly midway between point R and point G. Point K is 10 m to the South of point Q. Point L is 3 m towards the East of point Q. Point S is exactly midway between point G and point H.

**9.** If a persons walks 4 m towards the South from point L, takes a right turn and walks for another 3 m, then which of the following points would he reach?

(a) Q (b) G (c) K (d) H

**10.** If a person walks 8 m towards North from Points S, then which of the following points would he cross and how far will he be from point R?

(a) G, 4 m (b) H, 3 m (c) L, 6 m (d) L, 3 m

***Direction Q.(11 and 12) : In the following questions, the symbols %, □, $, # and @ are used with the following meaning as illustrated below:***

'P % Q' means 'P is neither smaller than nor equal to Q'.

'P □ Q' means 'P is not greater than Q'.

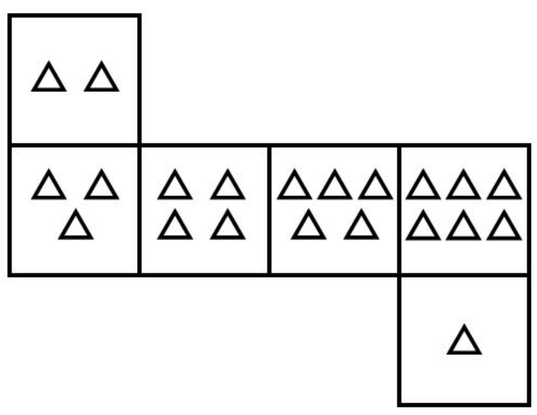
'P $ Q' means 'P is not smaller than Q'.

'P # Q' means 'P is neither greater than nor equal to Q'.
'P @ Q' means 'P is neither greater than nor smaller than Q'.

**11.** **Statements:** D □ R, R # M, M @ K, K % F
**Conclusions:**
I. F $ M II. K @ R III. K % R IV. D @ M
(a) Only I and II are true (b) Only (I) is true
(c) Only (II) is true (d) Only (III) is true

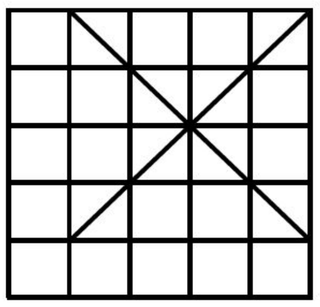
**12.** **Statements:** R $ B, B @ A, A % K, K # M
**Conclusions:**
I. M % A II. K $ B III. A % R IV. K # R
(a) Only (IV) is true
(b) Only (I) and (IV) are true
(c) Only (II) and (III) are true
(d) Only (I), (III) and (IV) are true

**13.** Which face will be at bottom when face with 5Δ's is at top?



(a) Δ (b) Δ Δ (c) Δ Δ Δ (d) Δ Δ Δ Δ

**14.** Find the number of squares and triangles.



(a) 30, 32 (b) 30, 24 (c) 55, 24 (d) 55, 48

**15.** Mohan correctly remembers that his father's birthday is before 20th January but after 16th January whereas his sister correctly remembers that their father's birthday is after 18th January but before 23rd January. On which date in January is definitely their father's birthday?
(a) 18th (b) 19th
(c) 20th (d) Data inadequate

**16.** Three statements are given followed by four conclusions (I), (II), (III) and (IV).
You have to consider the statements to be true even if they seem to be at variance from commonly known facts. You have to decide which of the given conclusions, if any, follows from the given statements.
**Statements:**
Some ice are rings.
No ring is paint.
Some rings are gold.
**Conclusions:**
I. No gold is paint. II. No ice is gold.
III. Some rings are paints. IV. All golds are rings.
(a) Only (I) and (III) follow
(b) Only (I) and (II) follow
(c) Only (III) and (IV) follow
(d) None of these

***Direction Q. (17 to 20) : Study the following information carefully and answer the questions given below:***
Seven people — N, K, T, B, M, W and R have their weekly offs on different days of the week *i.e.* Sunday, Monday, Tuesday, Wednesday, Thursday, Friday and Saturday, not necessarily in that order. Each of them has a liking for different cuisine *i.e.* Indian, Italian, Mexican, Chinese, Spanish, Continental and Thai, not necessarily in that order. K likes Thai food and gets his weekly off on Thursday. B likes Italian food and does not have off on Sunday. M has weekly off on Saturday and R has his weekly off on Tuesday. W likes continental food whereas the one who has weekly off on Monday likes Mexican cuisine. T does not like Spanish cuisine and has weekly off on Wednesday. The one who likes Indian food does not have a weekly off on Tuesday or Wednesday.

**17.** Who has a weekly off on Friday?
(a) T (b) R
(c) W (d) None of these

**18.** What cuisine does R like?
(a) Continental (b) Indian
(c) Italian (d) Spanish

**19.** On which day N have weekly off?
(a) Tuesday (b) Friday (c) Monday (d) Sunday

**20.** Who likes Chinese cuisine?
(a) T (b) B (c) R (d) N

## SOLUTIONS

**1. (d) :** –1 –1 –1 –1 –1 –1 –1 –1
Y X H G W V F E U T D C S R B A

**2. (b) :** The logic being followed is 1234 to 3412 and all capitals are converted into small.

**3. (a) :** 839 shall be read as 8 : 39 on the clock.
9 : 19 = 8 : 39 + 40 minutes.
∴ 10 : 52 = 10 : 12 + 40 minutes

**4. (b) :** Bottom row has all multiples of 20.
∴ ? = 60.

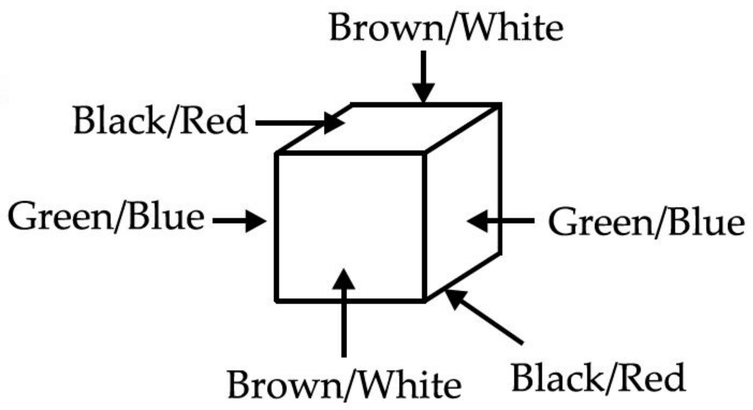
**5. (d) :** The given number ends with an even number and hence the last even number will be symbolised as ©.

| 6 | 7 | 3 | 2 | 5 | 8 |
|---|---|---|---|---|---|
| ↓ | ↓ | ↓ | ↓ | ↓ | ↓ |
| • | + | ? | * | $ | © |

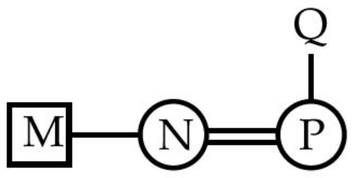
**6. (d) :**

| 2 | 3 | 6 | 4 | 7 | 5 |
|---|---|---|---|---|---|
| ↓ | ↓ | ↓ | ↓ | ↓ | ↓ |
| * | ? | • | ÷ | + | $ |

**7. (a) :**

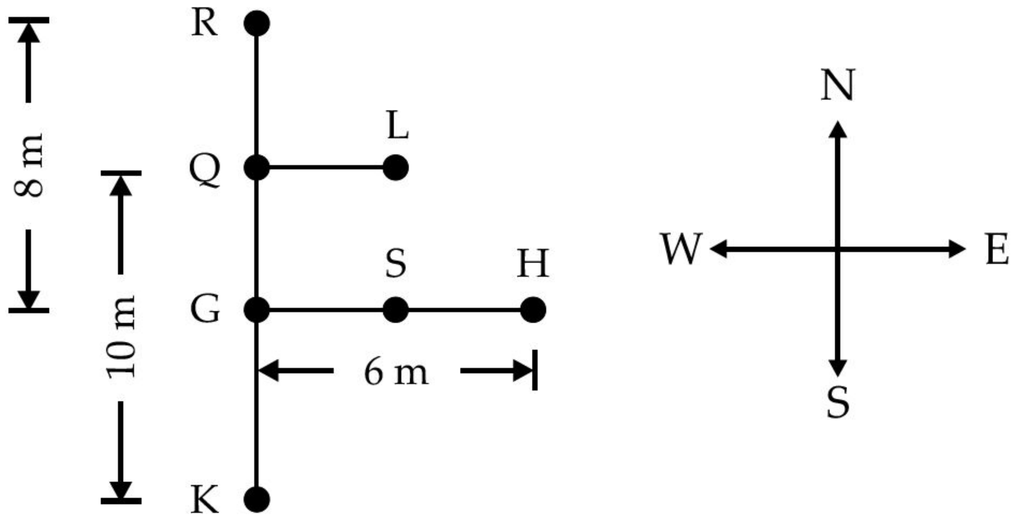



**8. (c) :** Statement: M + N – P × Q can be drawn as

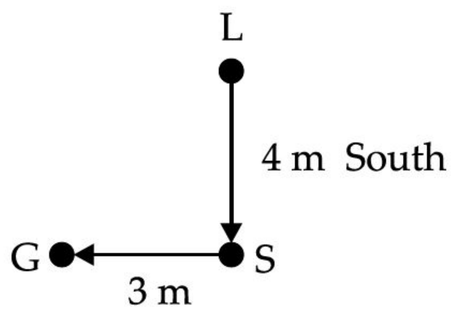



Thus, M is the son-in-law of Q.

**(9-10) :** According to the question the direction diagram will be as follows:

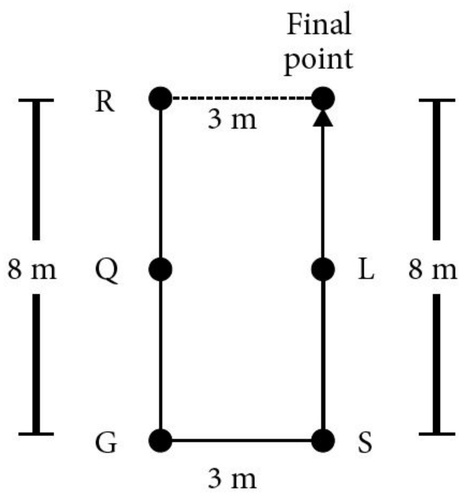



**9. (b) :**




Clearly, the person starts from L goes 4 m Southward and reaches point S from where he takes right turn and walks 3 m to reach G.

**10. (d) :**




Clearly, person will cross L and at the final point, he will be at a distance of 3 m from R.

**(11-12) :** (a) P % Q $\Rightarrow$ P $\not<$ Q; P $\neq$ Q

Therefore, P > Q

(b) P $\square$ Q $\Rightarrow$ P $\not>$ Q $\Rightarrow$ P $\leq$ Q

(c) P $ Q $\Rightarrow$ P $\not<$ Q $\Rightarrow$ P $\geq$ Q

(d) P # Q $\Rightarrow$ P $\not>$ Q ; P $\neq$ Q

Therefore, P < Q

(e) P @ Q $\Rightarrow$ P $\not>$ Q ; P $\not<$ Q

Therefore, P = Q

| % $\Rightarrow$ > | $\square$ $\Rightarrow$ $\leq$ | $ $\Rightarrow$ $\geq$ | # $\Rightarrow$ < | @ $\Rightarrow$ = |
|---|---|---|---|---|

**11. (d) : Statements:** D $\square$ R $\Rightarrow$ D $\leq$ R

R # M $\Rightarrow$ R < M

M @ K $\Rightarrow$ M = K

K % F $\Rightarrow$ K > F

Therefore, D $\leq$ R < M = K > F

**Conclusions:**

I. F $ M $\Rightarrow$ F $\geq$ M : Not true

II. K @ R $\Rightarrow$ K = R : Not true

III. K % R $\Rightarrow$ K > R : True

IV. D @ M $\Rightarrow$ D = M : Not true

Hence, only (III) is true.

**12. (a) : Statements:** R $ B $\Rightarrow$ R $\geq$ B

B @ A $\Rightarrow$ B = A

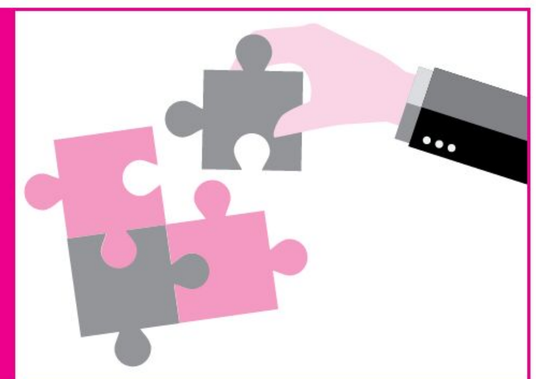
A % K $\Rightarrow$ A > K

K # M $\Rightarrow$ K < M

Therefore, R $\geq$ B = A > K < M

# PUZZLE CORNER

SOLUTION - JUNE 2025



| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 19+ 7 | 3 | 5− 1 | 6 | 4× 2 | 160× 4 | 8 | 5 |
| 4 | 5 | 15+ 8 | 7 | 1 | 2 | 2÷ 3 | 96× 6 |
| 13+ 1 | 7 | 5− 2 | 4 4 | 45× 5 | 3 | 6 | 8 |
| 5 | 2− 4 | 7 | 7− 8 | 3 | 5− 6 | 1 | 2 |
| 9+ 2 | 6 | 2− 5 | 1 | 1280× 8 | 11+ 7 | 4 | 84× 3 |
| 6 | 1 | 3 | 5 | 4 | 8 | 3− 2 | 7 |
| 384× 8 | 2 | 2÷ 6 | 3 | 42× 7 | 1 | 5 | 4 |
| 3 | 8 | 2÷ 4 | 2 | 6 | 35× 5 | 7 | 1 |

**Conclusions:**

I. M % A $\Rightarrow$ M > A : Not true

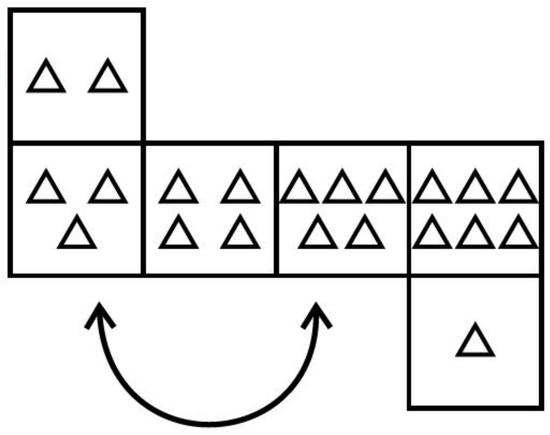
II. K $ B $\Rightarrow$ K $\geq$ B : Not true

III. A % R $\Rightarrow$ A > R : Not true

IV. K # R $\Rightarrow$ K < R : True

Hence, only (IV) is true.

**13. (c) :**

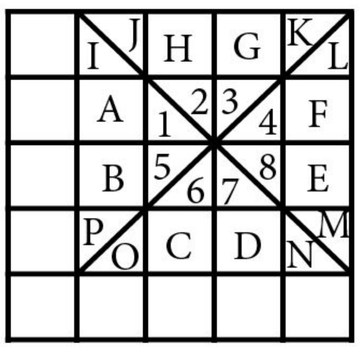


Opposite faces

By using standard result, we get 3Δ's is opposite to 5Δ's So, if face with 5Δ's is at top, the face with 3Δ's is at the bottom.

**14. (d) :** Number of squares in the given figure

$$=1^2+2^2+3^2+4^2+5^2=\frac{5\times6\times11}{6}=55$$




The given figure contain 48 triangles as :

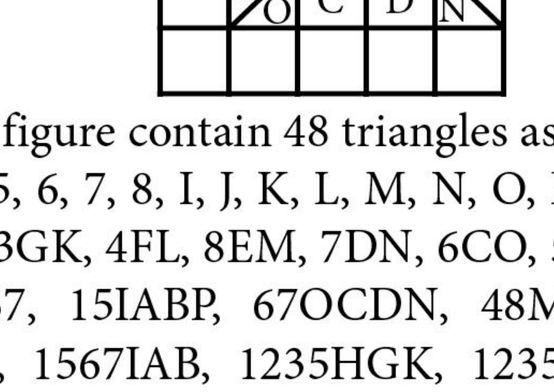
1, 2, 3, 4, 5, 6, 7, 8, I, J, K, L, M, N, O, P, 15, 67, 48, 23, 1AI, 2HJ, 3GK, 4FL, 8EM, 7DN, 6CO, 5BP, 1567, 1523, 2348, 4867, 15IABP, 67OCDN, 48MEFL, 23KGHJ, 1567CDN, 1567IAB, 1235HGK, 1235ABP, 4678EFL, 4678OCD, 2348JHG, 2348FEM, 1235ABPIJHGK, 2348JHGKLFEM, 4678OCDNMEFL, 1567IABPOCDN.

**15. (b) :** According to Mohan, his father's birthday may be on 17$^{th}$, 18$^{th}$ or 19$^{th}$ January.

According to his sister, their father's birthday may be on 19$^{th}$, 20$^{th}$, 21$^{st}$ or 22$^{nd}$ January.

Hence, the common date is 19$^{th}$ January.

**16. (d) :**




As gold may or may not be paint thus conclusion (I) is invalid. As ice may or may not be gold thus conclusion (II) is invalid. As no ring is a paint thus conclusion (III) is invalid. All gold may or may not be rings thus conclusion (IV) is invalid. Thus, none of the conclusion follows.

**(17 - 20) :**

| Person | Offs | Cuisine |
|---|---|---|
| N | Monday | Mexican Cuisine |
| K | Thursday | Thai Food |
| T | Wednesday | Chinese |
| B | Friday | Italian Food |
| M | Saturday | Indian Food |
| W | Sunday | Continental Food |
| R | Tuesday | Spanish Cuisine |

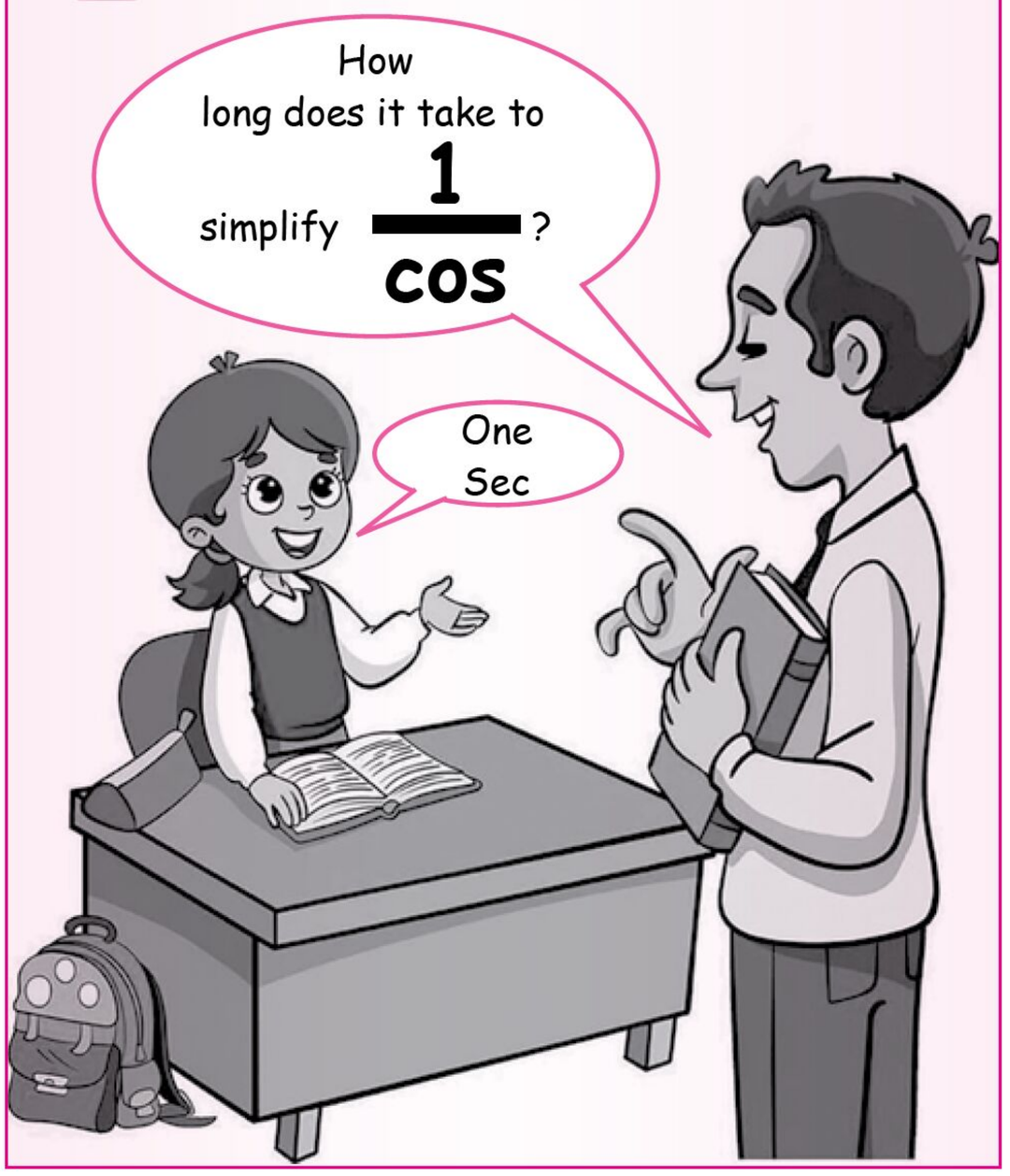
**17. (d) :** B has weekly off on Friday.

**18. (d) :** R likes Spanish cuisine.

**19. (c) :** N has weekly off on Monday.

**20. (a) :** T likes Chinese cuisine.

## COMIC CAPSULE



# QUANTITATIVE APTITUDE

## For Various Competitive Exams

**1.** Left pan of a faulty balance weighs 100 gram more than its right pan. A shopkeeper keeps the weight measure in the left pan while buying goods but keeps it in the right pan while selling his goods. He uses only 1 kg weight measure. If he sells his goods at the listed cost price, what is his gain?

(a) $\frac{200}{9}\%$ (b) $\frac{100}{11}\%$

(c) $\frac{100}{9}\%$ (d) $\frac{200}{11}\%$

**2.** Geeta, Shweta and Sita started a business with investment of ₹ 4200, ₹ 8400 and ₹ 5400 respectively. In the end of the years, there was a profit of ₹ 24000. Sita invested 32% of her share in profit in the business. The remaining amount of profit of Sita is

(a) ₹ 4986 (b) ₹ 4689 (c) ₹ 4886 (d) ₹ 4896

**3.** The diluted wine contains only 6 litres of wine and the rest is water. A new mixture whose concentration is 50% is to be formed by replacing with wine. How many litres of mixture be replaced with pure wine if there are initially 18 litres of water in the mixture?

(a) 5 litres (b) 6 litres (c) 8 litres (d) 12 litres

**4.** *A* can build a wall in 25 days and *B* can demolish the same wall in 80 days and *C* can build the wall in 60 days. If they work on consecutive days one after another starting from *A* on the first day. Then in how many days the work will be completed?

(a) 67 days (b) $67\frac{34}{48}$ days

(c) $66\frac{34}{48}$ days (d) $68\frac{34}{48}$ days

***Directions for questions 5 and 6: Answer the questions based on the following information:***

Sachin Tendulkar arranged a party for all his friends. There are 2000 friends invited for the party. In his party, among all the friends 48% preferred CSK, 54% like RCB and 64% SRH. Of the total, 28% liked CSK and RCB, 32% SRH and RCB and 30% CSK and SRH. Only 6% did not like any of these.

**5.** The ratio of the number of friends who like only CSK to the number who like only SRH is

(a) 2 : 3 (b) 3 : 2

(c) 5 : 3 (d) 2 : 5

**6.** The number of friends who like CSK and SRH but not RCB is

(a) 600 (b) 240

(c) 280 (d) 360

**7.** Four cards are drawn at random from a pack of 52 cards. Find the probability of getting all the four cards of different value.

(a) $\frac{157}{4165}$ (b) $\frac{119}{833}$

(c) $\frac{264}{4165}$ (d) $\frac{177}{4165}$

**8.** The number of ways of painting the faces of a cube with six different colors is

(a) 30 (b) 6

(c) 6! (d) 5!

**9.** A yearly payment to the servant is ₹ 90 plus one turban. The servant leaves the job after 9 months and receives ₹ 65 and a turban. Then the price of the turban is

(a) ₹ 10 (b) ₹ 15

(c) ₹ 7.50 (d) ₹ 5

**10.** Trains are moving from A to B and B to A at a regular interval of 1 hour ($24 \times 7$). They complete their journey in 5 hours. How many trains coming from station B will cross the train coming from station A that started at 10 A.M.? (Assume that the trains start from each of the station at the same time)

(a) 6 (b) 5 (c) 11 (d) 15

**11.** The last digit of the expression
$4 \times 9^2 \times 4^3 \times 9^4 \times 4^5 \times 9^6 \times ..... \times 4^{99} \times 9^{100}$ is
(a) 4 (b) 6 (c) 9 (d) 1

**12.** In fresh grapes, 70% of weight is because of water and in dried grapes, 40% weight is because of water. In a basket 60% of the weight is fresh grapes and rest are dried grapes. What is the percentage reduction in weight if all the fresh grapes turn into dried grapes?
(a) 30% (b) 50% (c) 40% (d) 20%

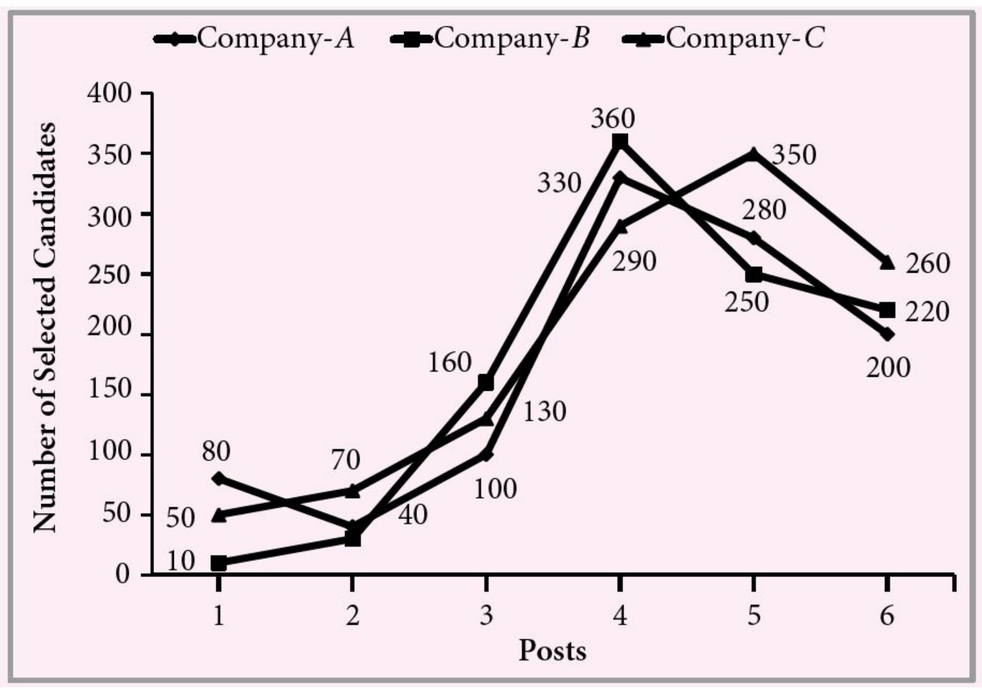
**13.** A trader owes a merchant ₹ 10028 due in 1 year, hence the trader wants to settle the account after 3 months. If the rate of interest is 12% per annum, how much cash should he pay?
(a) ₹ 9025 (b) ₹ 9220
(c) ₹ 9600 (d) ₹ 9500

**14.** The batting average of 40 innings of a cricket player is 50 runs. His highest score exceeds his lowest score by 172 runs. If these two innings are excluded the average of the remaining 38 innings is 48. His highest score was
(a) 172 (b) 173
(c) 174 (d) 176

**15.** A man rows a distance of 24 km in 1 hour in still water and in 80 minutes against the current. The time taken by him to row 84 km with the current and return to his starting point is
(a) 14 hours (b) 12 hours
(c) 16 hours (d) 4 hours

***Directions for questions 16 to 20: Answer the questions on the basis of the information given below:***

In the following graph, the number of selected candidates for 6 different posts by three different companies A, B and C has been shown.




POST 1 : HR Officer
POST 2 : IT Officer
POST 3 : Assistant Manager
POST 4 : Advertisement Office Assistant
POST 5 : Office Assistant Operations
POST 6 : Junior Office Administrator

**16.** What is the ratio between the number of all candidates selected for company *A* and that selected for the posts of assistant managers and junior office administrators in all three companies?
(a) 103 : 107 (b) 102 : 107
(c) 103 : 106 (d) 113 : 117

**17.** The number of candidates recruited for the post of office assistant operations in company *B* is approximately what percent of total candidates recruited in that company?
(a) 28% (b) 24%
(c) 30% (d) 31%

**18.** The number of candidates recruited for the posts of assistant manager and advertisement office assistant is what percent of the candidates recruited for the post of junior office administrator and HR officer by the company *C*?
(a) 115% (b) 120%
(c) 135% (d) 141%

**19.** The total number of candidates recruited for the post of HR officers in all the companies is what percent of the total candidates recruited by the company *A* for all posts?
(a) 16% (b) 11%
(c) 12% (d) 14%

**20.** What is the respective ratio between the number of candidates selected for all the posts by company *A* and company *C*?
(a) 113 : 115 (b) 115 : 113
(c) 113 : 117 (d) 103: 115

## SOLUTIONS

**1. (a) :** Let the price of 1 g be ₹ 1.
The shopkeeper buys 1100 g at the rate of ₹ 1000
He sells 900 g at the cost price of ₹ 1000.

So, CP for 900 g would be $= \frac{1000}{1100} \times 900 = ₹\ 818.18$

So, gain % $= \frac{1000 - 818.18}{818.18} \times 100 = 22.22\% = \frac{200}{9}\%$

**2. (d) :** As, the time they invested is same, hence profit % is same as the ratio of amount.

*i.e.* 4200 : 8400 : 5400 = 42 : 84: 54 = 7 : 14 : 9

$$\text{Profit of Sita} = \frac{9}{30} \times ₹\,24000 = ₹\,7200$$

$\therefore$ Remaining profit of Sita = ₹ 7200 × 0.68 = ₹ 4896

**3. (c) :**

| **Wine** | **Water** | |
|---|---|---|
| 6 litres | 18 litres | |
| 1 | 3 | |
| 25% | 75% | (original ratio) |
| 50% | 50% | (required ratio) |

In this case, the percentage of water is being reduced when the mixture is being replaced with wine.

So, the ratio of left quantity to the initial quantity is 2 : 3.

Therefore, $\frac{2}{3} = \left[1 - \frac{k}{24}\right] \Rightarrow \frac{k}{24} = \frac{1}{3} \quad \Rightarrow \quad k = 8$ litres

**4. (c) :** Let the total work be 1200 units [LCM (25, 80, 60)]

$$A\text{'s one day work} = \frac{1200}{25} = 48 \text{ units}$$

$$B\text{'s one day work} = \frac{1200}{80} = -15 \text{ units}$$

$$C\text{'s one day work} = \frac{1200}{60} = 20 \text{ units}$$

First 3 days work = (48 + 20 − 15) = 53 units

Now, on $66^{th}$ day = 53 × 22

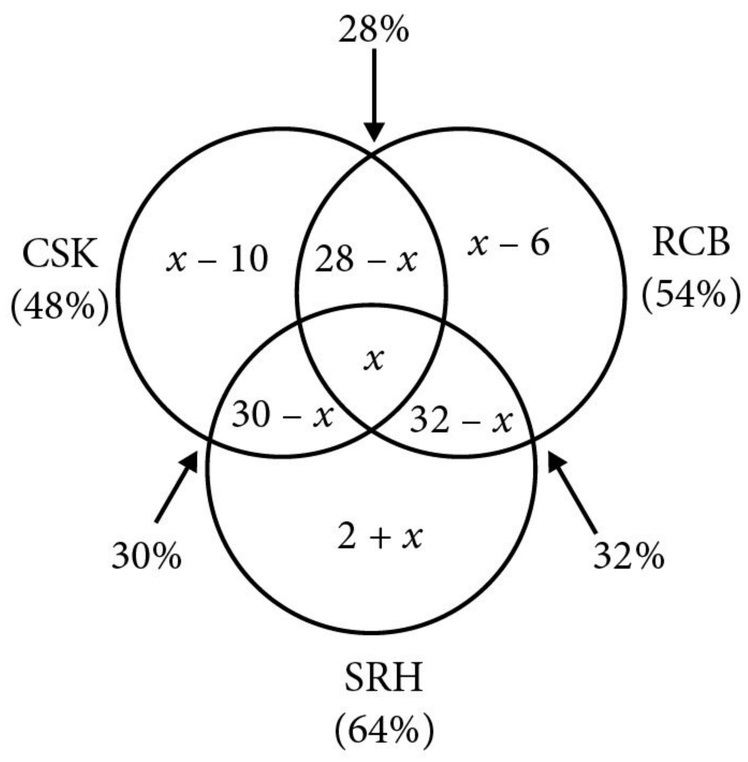
= 1166 units be completed.

Work left = 1200 − 1166 = 34 units

On $67^{th}$ day the work will be completed by $A = \frac{34}{48}$ units

So, the work will be completed in $66\frac{34}{48}$ days .

**(5 - 6) :**




Given, Not like any team = 6%

$\Rightarrow$ 94% = $x - 10 + x - 6 + x + 2 + 28 - x + 30 - x + 32 - x + x$

$\Rightarrow$ $x = 18\%$

**5. (d) :** $\frac{x-10}{2+x} = \frac{18-10}{18+2} = \frac{8}{20} = \frac{2}{5}$

**6. (b) :** 30 − $x$ = 30 − 18 = 12%

*i.e.* 12% of 2000 = 240

**7. (c) :** $n(S) = {}^{52}C_4 = 270725$

Favourable number of outcomes = ${}^{13}C_1 \times {}^{12}C_1 \times {}^{11}C_1 \times {}^{10}C_1$

= 13 × 12 × 11 × 10

$\therefore$ $P$(different value on all cards)

$$= \frac{13 \times 12 \times 11 \times 10}{270725} = \frac{264}{4165}$$

# SAMURAI SUDOKU

Samurai Sudoku puzzle consists of five overlapping sudoku grids. The standard sudoku rules apply to each 9 × 9 grid. Place digits from 1 to 9 in each empty cell. Every row, every column and every 3 × 3 box should contain one of each digit.

The puzzle has a unique answer.

| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | 5 | | 9 | 1 | | | | | | 4 | | 7 | | | 6 | | |
| | 8 | | 6 | | | | | | | | | 9 | | | 8 | | | | | |
| 9 | 7 | | | | | | | | | | | | | | 2 | | | | 7 | |
| | 9 | 5 | 1 | | | 7 | 6 | | | | | 5 | | | | | | | 2 | |
| 3 | | | | 5 | | | 2 | | | | | | | | | 6 | | | 9 | 4 |
| | | | 7 | | | | 9 | | | | | | | | | 7 | | 3 | 6 | |
| 2 | | | | 9 | | | 8 | | | 5 | | 3 | | | 5 | 4 | 9 | | 8 | |
| | | 6 | 2 | | | 5 | | 9 | | | | | 7 | 6 | 3 | | | 4 | | |
| | | | | 7 | | 3 | | | 6 | | | 4 | | | | | | | | 1 |
| | | | | | | | | | | 3 | | 2 | | 8 | | | | | | |
| | | | | | | 4 | | | 9 | | | | | | | | | | | |
| | | | | | | 7 | | | | | | | | | | | | | | |
| 1 | 8 | | 6 | | | 9 | 3 | | | | | 1 | 2 | | | | | 9 | | |
| | | | | | | 2 | | 5 | 7 | | | | 8 | | | | 3 | | | |
| 5 | | | | 3 | 9 | | 7 | | | | | | | 5 | | 7 | | | | |
| 4 | | | 3 | | | | 9 | | | | | 4 | | 8 | 9 | 5 | | 2 | | |
| | | | | | 5 | 1 | | | | | | 7 | | | 4 | | | | | |
| 9 | | | | | | 3 | | | | | | | | 3 | | | | 6 | | |
| 7 | 5 | 2 | | | | 6 | | | | | | 5 | | 2 | 6 | | | 3 | 7 | 1 |
| | 4 | | 9 | 1 | | | | | | | | | 9 | | 3 | | | | 4 | 2 |
| | | | | | 7 | | | | | | | | | | 2 | | | | | |

*Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.*

**8. (a) :** Coloring the first face have 1 way, since all faces are same.

Now, painting opposite face of first face have 5 ways.

Rest four faces can be treated as a circular arrangement *i.e*, (4 – 1)! ways.

So, total number of arrangements = $1 \times 5 \times 3$ !

$= 5 \times 6 = 30$ ways.

**9. (a) :** Let the cost of the turban be $T$.

Hence, total payment for one *y*ear = ₹ $90 + T$.

So, the payment for 9 months should be

$$\frac{3}{4}(90 + T) = (65 + T) \qquad \text{(Given)}$$

$$\Rightarrow \quad T = ₹\ 10$$

**10. (c) :** The train which starts from $B$ at 10 A.M. will definitely cross the train starting from $A$.

Also, 5 trains starting from $B$ before 10 A.M. and 5 trains starting from $B$ after 10 A.M. will also cross.

So, required number of trains = 5 + 5 + 1 = 11

**11. (b) :** The unit digit of each pair is 4, and there are 50 such pairs which are multiplied together.

$$\underset{\downarrow}{\boxed{4 \times 9^2}} \times \underset{\downarrow}{\boxed{4^3 \times 9^4}} \times \underset{\downarrow}{\boxed{4^5 \times 9^6}} \times .. \times \underset{\downarrow}{\boxed{4^{99} \times 9^{100}}}$$

$$4 \qquad\qquad 4 \qquad\qquad 4 \qquad\qquad 4$$

$= 4 \times 4 \times 4 \times 4$ ..... 4 (upto 50 times)

*i.e.*, the unit digit of $4^{50}$, which is 6.

[Since unit digit of $4^{2n}$ is 6 for $n = 1, 2, 3,$ .... etc.]

Thus, finally we get 6 as unit digit.

**12. (a) :** When fresh grapes turns into dry grapes the weight reduces to $\frac{1}{2}$ of the original weight (since pulp that consists 30% is now 60% of the weight in dried grapes).

So, the final weight as $x$ % of the original weight be

$$= \frac{1}{2}(60\%) + (40\%) = 70\%$$

Loss or reduction of weight = (100 – 70)% = 30%

**13. (b) :** Let the present value of money be $x$.

Then, $\frac{x \times 12}{100} + x = 10028 \quad \Rightarrow \quad 0.12x + x = 10028$

$$\Rightarrow \quad x = \frac{10028}{1.12}$$

This amount after 3 months

$$= \frac{\frac{10028}{1.12} \times 12 \times 3}{12 \times 100} + \frac{10028}{1.12} = \frac{10028 \times 3}{1.12 \times 100} + \frac{10028}{1.12}$$

$$= \frac{10028 \times 103}{112} = 9222.17 \approx ₹\ 9220$$

**14. (c) :** Total score in 40 innings = $40 \times 50 = 2000$

Total score in 38 innings = $38 \times 48 = 1824$

$\therefore$ Sum of highest and lowest score, *i.e.*, $x$ and $y$ is,

$x + y = 2000 - 1824 = 176$ ...(i)

and given, $x - y = 172$ ...(ii)

Solving equation (i) and (ii), we get

$x = 174$

**15. (c) :** Speed in still water = $\frac{24}{1} = 24$ km / h

$\therefore$ Speed against the current = $\frac{24}{4/3} = 18$ km / h

[$\because$ 80 minutes = $\frac{4}{3}$ hours ]

Therefore, upstream speed = 24 – 18 = 6 km/h

Downstream speed = 24 + 18 = 42 km/h

Hence, to row 84 km with the current and return to starting point it will take $\frac{84}{42} + \frac{84}{6} = 2 + 14 = 16$ hours

So, he will take 16 hours.

**16. (a) :** Total number of candidates selected for company-$A$ = 80 + 40 + 100 + 330 + 280 + 200 = 1030

and (Assistant Managers + Junior Office Administrators)

= (100 + 130 + 160) + (200 + 220 + 260)

= 390 + 680 = 1070

$\therefore$ Required ratio = 1030 : 1070 = 103 : 107

**17. (b) :** Total candidates selected in Company-$B$

= 10 + 40 + 160 + 360 + 250 + 220 = 1040

$\therefore$ Required percentage $= \frac{250}{1040} \times 100 \approx 24\%$

**18. (c) :** Company-$C$

(Assistant Manager + Advertisement Office Assistant)

= 130 + 290 = 420

(Junior Office Administrator + HR Officer)

= 260 + 50 = 310

$\therefore$ Required percentage $= \frac{420}{310} \times 100 \approx 135\%$

**19. (d) :** Total number of HR Officers selected

= 80 + 10 + 50 = 140

$\therefore$ Required percentage $= \frac{140}{1030} \times 100 \approx 14\%$

**20. (d) :** Total candidates selected:

Company-$A \Rightarrow$ 1030

Company-$C \Rightarrow$ 1150

$\therefore$ Required ratio = 1030 : 1150 = 103 : 115

# GK CORNER

Enhance Your General Knowledge with Current Updates!

## SPORTS

**1** On May 22, 2025, England's star batsman, Joe Root etched his name in cricketing history by becoming the fastest player to reach 13,000 runs in Test cricket. Root achieved this milestone during his 153rd Test match in a one-off Test against Zimbabwe at Trent Bridge by surpassing the previous record held by South African legend, Jacques Kallis. Root solidified his place as one of the game's greats.

**2** Neeraj Chopra made history at the Doha Diamond League on May 16, 2025 becoming the first Indian javelin thrower to surpass the 90-meter mark, recording a personal best of 90.23 meters throw. This remarkable achievement made him the first Indian and the third Asian ever, placing him among an elite group of 25 athletes worldwide.

**3** The Mumbai Cricket Association (MCA) honoured Indian cricket legend, Rohit Sharma at Wankhede Stadium by renaming the Divecha Pavilion Level 3 stand as the "Rohit Sharma Stand," following his recent retirement from Test and T20 formats. The MCA also unveiled stands named after former Indian captain, Ajit Wadekar and veteran cricket administrator, Sharad Pawar, while dedicating the MCA Office Lounge to former MCA president, Amol Kale.

**4** FIFA has lifted the suspension on the Republic of Congo's football federation, FECOFOOT, bringing an end to a three-month standoff caused by third-party interference, primarily involving the Ministry of Sports in Congo. The suspension has hurt the day-to-day activities of the federation, including the shutdown of its headquarters as well as several training centers. Congolese Sports Minister, Hugues Ngouelondele initially denied interference, but later sought negotiations with FIFA.

**5** Pankaj Advani, claimed his third CCI Billiards Classic title with a 5-2 victory over Dhruv Sitwala in the final held at the Cricket Club of India (CCI), Mumbai.

**6** Miss Estonia Eliise Randmaa made history by securing gold in the Miss World 2025 Sports Challenge held at the Gachibowli Indoor Stadium, Hyderabad. Her victory secured Estonia's place in the quarter-finals of the Miss World pageant for the first time since 1999.

**7** India has been awarded the hosting rights for the International Shooting Sport Federation (ISSF) Junior World Cup Rifle/Pistol/shotgun 2025, which will mark a milestone for Indian shooting sports, showcasing the country's growing prominence in the global arena. India has previously hosted major ISSF events, including two World Cup Finals and four senior World Cups.

**MONTHLY TEST DRIVE CLASS XII ANSWER KEY**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | (b) | 2. | (d) | 3. | (c) | 4. | (b) | 5. | (c) |
| 6. | (c) | 7. | (a,b,c,d) | 8. | (a,b) | 9. | (a,b,c,d) | 10. | (b,c) |
| 11. | (a,c) | 12. | (a,b) | 13. | (a,b) | 14. | (d) | 15. | (b) |
| 16. | (b) | 17. | (3) | 18. | (3) | 19. | (0) | 20. | (1) |

8 The 25th edition of the Fit India Sundays on cycle initiative saw our 15,000 participants join the Tiranga Cycling Rally on June 1, 2025. The event celerbrated fitness and paid tribute to India's armed forces, blending national pride with a health-first message.

9 India won 10 medals at the 2025 Senior Asian Wrestling Championship held in Amman, Jordan – including 1 gold by Manisha Bhanwala and silvers by olympian Deepak Punia and Udit. Team showcased strong performances across multiple weight categories, securing 3 silver and 7 bronze medals overall.

10 Nepali mountaineer, Kami Rita Sherpa made history yet again by summiting Mount Everest for a record 31st time on May 27, 2025. Leading a 22-member Indian Army team via the Southeast ridge, his feat highlights the unmatched legacy of sherpas in high-altitude climbing.

## Test Yourself!

1. Who became the fastest player to reach 13,000 runs in Test cricket?
   (a) Sachin Tendulkar (b) Joe Root
   (c) Jacques Kallis (d) Virat Kohli

2. Besides Rohit Sharma, which former Indian cricket captain had a stand named after him by the MCA?
   (a) Kapil Dev (b) Ajit Wadekar
   (c) Rahul Dravid (d) Sourav Ganguly

3. At which event did Neeraj Chopra surpass the 90-meter mark in javelin throw for the first time?
   (a) 2025 Tokyo Olympics
   (b) 2025 Doha Diamond League
   (c) 2025 Commonwealth Games
   (d) 2025 Asian Games

4. Miss World 2025 Sports Challenge was held at which stadium?
   (a) Wankhede Stadium
   (b) Arun Jaitley Stadium
   (c) Gachibowli Indoor Stadium
   (d) Eden Gardens

5. Which country's football federation had its FIFA suspension lifted in 2025 after resolving third-party interference issues?
   (a) Republic of Congo (b) Norway
   (c) Nigeria (d) Ghana

6. Who did Pankaj Advani defeat to win his third CCI Billiards Classic title in 2025?
   (a) Sourav Kothari
   (b) Geet Sethi
   (c) Aditya Mehta
   (d) Dhruv Sitwala

7. How many medals did India won at the 2025 Senior Asian Wrestling Championship in Amman?
   (a) 5 (b) 8
   (c) 10 (d) 12

8. What was the primary purpose of the Tiranga Cycling Rally held on June 1, 2025?
   (a) To break a world cycling record
   (b) To promote eco-frindely transport
   (c) To celebrate fitness and honour India's armed forces.
   (d) To raise funds for cycling development

9. Which group did Kami Rita Sherpa led during his 31st Mount Everest ascent?
   (a) A team of international climbers
   (b) A group of Sherpa trainees
   (c) A 22-member team from the Indian Army
   (d) A Nepalese scientific expedition

10. Which major shooting event will India host in 2025?
    (a) ISSF Senior World Cup
    (b) ISSF Junior World Cup
    (c) Common Wealth Shooting Championship
    (d) Asian Air Gun Championship

## Answer Key

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. (b) | 2. (b) | 3. (b) | 4. (c) | 5. (a) | 6. (d) | 7. (c) | 8. (c) | 9. (c) | 10. (b) |

# YOU ASK WE ANSWER

**Do you have a question that you just can't get answered?**

Use the vast expertise of our MTG team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough.

The best questions and their solutions will be printed in this column each month.

**1.** In a soccer tournament eight teams play each other once, with two points awarded for a win, one point for a draw and zero for a loss. How many points must a team score to ensure that it is in the top four (*i.e.* has more points than at least four other teams)?. ***(Ruhi, U.P.)***

**Ans.** Since, there are 8 teams, there are 7 rounds of four matches and thus a total of $7 \times 8 = 56$ points available.

Consider a team with 10 points. It is possible to have 5 teams on 10 points and 3 teams on 2 points when each of the top 5 draws with each other, each of the bottom 3 draws with each other and each of the top 5 wins against each of the bottom 3. So, 10 points does not guarantee a place in the top 4.

Consider a team with 11 points. If this team was fifth, then the number of points gained by the top 5 teams is $\geq 55$. This is impossible as the number of points shared by the bottom 3 teams is then 1, as these 3 teams must have at least $3 \times 2 = 6$ points between them for the games played between themselves. Hence, 11 points is sufficient to ensure a place in the top 4. Thus 11 points are required.

**2.** Let $B_1$ and $C_1$ be points on the sides $AC$ and $AB$ of triangle $ABC$. Lines $BB_1$ and $CC_1$ intersect at point $D$. Prove that a circle can be inscribed inside quadrilateral $AB_1DC_1$ if and only if the incircles of the triangles $ABD$ and $ACD$ are tangent to each other. ***(Sajjad, Mumbai)***

**Ans.** Let the incircle of triangle $ABD$ be tangent to $\overline{AD}$ at $T_1$ and let the incircle of triangle $ACD$ be tangent to $\overline{AD}$ at $T_2$. Then,

$$DT_1 = \frac{1}{2}(DA + DB - AB)$$

$$\text{and } DT_2 = \frac{1}{2}(DA + DC - AC)$$

First suppose a circle can be inscribed inside $AB_1DC_1$. Let it be tangent to sides $AB_1$, $B_1D$, $DC_1$, $C_1A$ at points $E, F, G, H$ respectively. Using equal tangents, we have

$AB - BD = (AH + HB) - (BF - DF)$

$= (AH + BF) - (BF - DF) = AH + DF$

and similarly $AC - CD = AE + DG$.

Also, $AH + DF = AE + DG$ by equal tangents, implying that $AB - BD = AC - CD$ and thus $DA + DB - AB = DA + DC - AC$. Therefore $DT_1 = DT_2$, $T_1 = T_2$, and the two given incircles are tangent to each other.

Next suppose the two incircles are tangent to each other. Then, $DA + DB - AB = DA + DC - AC$. Let $\omega$ be the incircle of $ABB_1$, and let $D'$ be the point on $\overline{BB_1}$ (different from $B_1$) such that line $CD'$ is tangent to $\omega$. Suppose, by way of contradiction, that $D \neq D'$. From the result in the last paragraph, we know that the incircles of triangles $ABD'$ and $ACD'$ are tangent and hence $D'A + D'B - AB = D'A + D'C - AC$. Because $DB - AB = DC - AC$ and $D'B - AB = D'C - AC$, we must have $DB - D'B = DC = D'C$ by subtraction. Thus, $DD' = |DB - D'B| = |DC - D'C|$. The triangle inequality then fails in triangle $DD'C$, a contradiction. This completes the proof.

**3.** Let $m, n$ be positive integers such that $m \leq \frac{n^2}{4}$ and every prime divisor of $m$ is less than or equal to $n$. Show that $m$ divides $n!$. ***(Neha, Delhi)***

**Ans.** It suffices to show that $p^k n!$ for all primes $p$ and integers $k \geq 1$ such that $p^k m$. If $k = 1$, we are done because we are given that $p \leq n$ and hence $p|n!$.

Otherwise, because $m \leq \frac{n^2}{4}$, we have $p^k \leq \frac{n^2}{4}$.

Thus, $n \geq 2\sqrt{p^k}$.

If $n \geq kp$, then at least $k$ of the numbers $1, 2, \ldots, n$ are multiples of $p$, implying that $p^k|n!$. Hence, it suffices to show that $2\frac{\sqrt{p^k}}{p} \geq k$ or equivalently that

$p^{\frac{(k-2)}{2}} \geq \frac{k}{2}$ ...(i), because this implies $n \geq 2\sqrt{p^k} \geq kp$.

Indeed, we can do so for most values of $p$ and $k$. If $k = 2$, then (i) reads $1 \geq 1$. If $k \geq 4$, then Bernoulli's inequality implies that $p^{\frac{(k-2)}{2}} = \left(1 + (p-1)\right)^{\frac{(k-2)}{2}} \geq 1 + \frac{k-2}{2}(p-1) \geq \frac{k}{2}$, as desired. Finally, if $k = 3$, then (i) is clearly true unless $p = 2$.

If indeed $k = 3$ and $p = 2$, then $m \geq 8$, $n \geq 6$ and $n!$ is certainly divisible by 8. Thus, we have shown that $p^k|n!$ in all cases, completing the proof. ◈◈

Infinite Learning

# Boundaries, Balance, and Brilliance: The Inequality Equation

When you hear the word "Inequality", your mind might leap to Economics or Social justice–but in the world of Mathematics, Linear Inequalities open the door to a different kind of world. Whether you're budgeting, planning, or analysing data, this concept forms the backbone of many real-world decisions. Mastering this topic equips students to interpret real-life scenarios through a mathematical lens.

## From Equations to Inequalities

At its core, a linear inequality is just like a linear equation, but instead of stating that two sides are equal, it gives us flexibility, modelling ranges, limits, and tells us that one side is less than (<), greater than (>), less than or equal to (≤), and greater than or equal to (≥) other. Behind these symbols lies a toolkit for solving real-world problems.

Consider, a situation where x and y are two products that are produced, and 100 is your budget. This will be represented as, $5x+2y \leq 100$.

This isn't just a string of numbers and symbols; it represents all values of x and y that make this statement true.

## What Linear Inequalities Teach Us?

Even beyond their direct applications, learning linear inequalities nurtures some of the most valuable 21st-century skills:

- **Decision-Making Under Constraints:** Life constantly asks us to choose under pressure–limited time, resources, or options. Inequalities model this.
- **Optimization Thinking:** How can we do the most with what we have? How do we stay within boundaries but get maximum results?
- **Visual Problem Solving:** Graphing inequalities helps students–and professionals–see problems spatially and visually, making abstract ideas concrete.

## From Classrooms to Careers

Here's how various professions use linear inequalities, often without calling them by name:

| Profession | Real-World Use of Inequalities |
|---|---|
| Economist | Modelling supply/demand constraints in market analysis |
| Engineer | Keeping forces, stress, and load within safe ranges |
| Software Developer | Creating logic-based conditions in code |
| Logistics Manager | Allocating delivery routes within time/fuel limits |
| Dietitian | Planning meals within calorie, fat, or carb restrictions |
| Environmental Scientist | Ensuring emissions stay below legal thresholds |

## Applications of Linear Inequalities

1. **Business:** Companies use linear programming, which relies on systems of inequalities, to maximize profits or minimize costs while staying within resource limits.
2. **Engineering:** Ensure stress, load, or voltage remains under safe limits.
3. **Health & Fitness:** Balance calorie intake and workout plans to meet goals.
4. **Programming & AI:** Define constraints in optimization problems.

## Conclusion

Linear inequalities may begin as algebraic expressions on a chalkboard, but their influence reaches far beyond the academic realm. They equip us with the ability to navigate limitations, analyse constraints, and evaluate feasible solutions. Whether applied in engineering, economics, data science, or daily decision-making, linear inequalities cultivate a mindset rooted in flexibility, critical thinking, and strategic planning. Mastering this concept is not merely about solving math problems–it's about developing the intellectual tools to make realistic decisions in complex environments. In embracing the language of inequalities, we learn not just how to calculate, but how to adapt, optimize, and thrive within boundaries.

# CBSE

# Warm-up!

CLASS-XI

*Chapterwise practice questions for CBSE Exams as per the latest pattern and syllabus by CBSE for the academic session 2025-26.*

## Series-3 Trigonometric Functions

***General Instructions : Read the following instructions very carefully and strictly follow them :***

(i) This question paper contains **38** questions. **All** questions are **compulsory**.

(ii) This question paper is divided into **five** Sections – **A, B, C, D** and **E**.

(iii) In **Section A**, Questions no. **1** to **18** are multiple choice questions (MCQs) and Questions no. **19 and 20** are Assertion-Reason based questions of **1** mark each.

(iv) In **Section B**, Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.

(v) In **Section C**, Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.

(vi) In **Section D**, Questions no. **32** to **35** are long answer (LA) type questions, carrying **5** marks each.

(vii) In **Section E**, Questions no. **36** to **38** are case study based questions, carrying **4** marks each.

(viii) There is no overall choice. However, an internal choice has been provided in **2** questions in Section **B**, **3** questions in Section **C**, **2** questions in Section **D** and **2** questions in Section E.

(ix) Use of calculators is **NOT** allowed.

**Time Allowed :** 3 hours **Maximum Marks :** 80

### SECTION - A

**This section comprises 20 Multiple Choice Questions (MCQs) of 1 mark each.** **20 × 1 = 20**

**1.** Convert 40°20′ into radian measure.

(a) 111 π/540 (b) 121 π/540

(c) 131 π/540 (d) 128 π/541

**2.** A circular wire of radius 7 cm is cut and bend again into an arc of a circle of radius 12 cm. The angle subtended by the arc at the centre is

(a) 50° (b) 210° (c) 100° (d) 60°

**3.** The circular measure of 75° angle is

(a) $\frac{\pi}{3}$ (b) $\frac{5\pi}{12}$ (c) $\frac{3\pi}{4}$ (d) $\frac{2\pi}{7}$

**4.** Find the degree measure of $\left(\frac{\pi}{9}\right)$ radian and $\left(-\frac{\pi}{3}\right)$ radian respectively.

(a) 20°, –60° (b) 30°, 60°

(c) 20°, 40° (d) 40°, 50°

**5.** If $\frac{\sin x}{\cos x} \times \frac{\sec x}{\operatorname{cosec} x} \times \frac{\tan x}{\cot x} = 9$, where $x \in \left(0, \frac{\pi}{2}\right)$, then the value of $x$ is equal to

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\pi$

**6.** If $\sin A - \sqrt{6}\cos A = \sqrt{7}\cos A$, then $\cos A + \sqrt{6}\sin A$ is equal to

(a) $\sqrt{6}\sin A$ (b) $\sqrt{7}\sin A$

(c) $\sqrt{6}\cos A$ (d) $\sqrt{7}\cos A$

**7.** If $3\sin\theta + 5\cos\theta = 4$, then the value of $5\sin\theta - 3\cos\theta$ is

(a) $\pm 3\sqrt{2}$ (b) $\pm 5\sqrt{2}$ (c) $\pm\sqrt{2}$ (d) $\pm 7\sqrt{2}$

**8.** If $x \sin 45° \cos^2 60° = \frac{\tan^2 60° \operatorname{cosec} 30°}{\sec 45° \cot^2 30°}$, then $x =$

(a) 2 (b) 4 (c) 8 (d) 16

**9.** If $\cot x = 4/3$ and $x$ lies in third quadrant, then find the value of $\sec x$ is

(a) 1/4 (b) 7/4 (c) 2/4 (d) –5/4

**10.** The value of $\sqrt{2}\sin\left(\frac{\pi}{4}-\theta\right)$ is

(a) $-(\sin\theta+\cos\theta)$ (b) $\sin\theta-\cos\theta$

(c) $\cos\theta+\sin\theta$ (d) $\cos\theta-\sin\theta$

**11.** The value of $\cos 80^\circ\cos 20^\circ + \sin 80^\circ \sin 20^\circ$ is

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{2}{\sqrt{3}}$ (c) 2 (d) $\frac{1}{2}$

**12.** The value of $\sin^2 17.5^\circ + \sin^2 72.5^\circ$ is equal to

(a) $\cos^2 90^\circ$ (b) $\tan^2 45^\circ$ (c) $\cos^2 30^\circ$ (d) $\sin^2 45^\circ$

**13.** The value of $\sec\frac{2\pi}{3}+\text{cosec}\frac{5\pi}{6}$ is equal to

(a) 2 (b) –2 (c) 4 (d) 0

**14.** Find the value of

$\sin(40^\circ+\theta)\cos(10^\circ+\theta)-\cos(40^\circ+\theta)\sin(10^\circ+\theta)$.

(a) 1/2 (b) 1/4 (c) 1 (d) 0

**15.** The degree measure of $\frac{11}{15}$ radian is

(a) $50^\circ$ (b) $42^\circ$ (c) $20^\circ$ (d) $10^\circ$

**16.** The value of $\frac{\sin(90^\circ+A)}{\cos(-A)}-\frac{\sin(180^\circ-A)}{\sin(-A)}+\frac{\tan(270^\circ+A)}{\cot(-A)}$ is

(a) – 1 (b) 3 (c) –3 (d) 1

**17.** Convert the angle $50^\circ\,37'\,30''$ in radian measure.

(a) $\frac{7\pi}{32}$ (b) $\frac{5\pi}{32}$ (c) $\frac{9\pi}{32}$ (d) $\frac{\pi}{32}$

**18.** The value of $4\sin A\cos^3 A - 4\cos A\sin^3 A$ is equal to

(a) $\cos 2A$ (b) $\sin 3A$ (c) $\sin 2A$ (d) $\sin 4A$

**Questions number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) options as given below.**

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is **not** the correct explanation of the Assertion (A).

(c) Assertion (A) is true, but Reason (R) is false.

(d) Assertion (A) is false, but Reason (R) is true.

**19. Assertion (A) :** The perimeter of a certain sector of a circle is equal to the length of the arc of the semi-circle. Then the angle at the centre of the sector in radians is $\frac{\pi}{2}$.

**Reason (R) :** Radian measure $= \frac{\pi}{180}\times$ Degree measure.

**20. Assertion (A) :** The smallest value of $5\cos\theta + 12$ is 7.

**Reason (R) :** Range of $\cos x$ is $[-1, 1]$.

## SECTION - B

**This section comprises of 5 Very Short Answer (VSA) type questions of 2 marks each.** **5 × 2 = 10**

**21.** The minute hand of a watch is 1.5 cm long. How far does its tip move in 40 minutes? (Use $\pi = 3.14$)

**22. (a)** If $A + B = \frac{5\pi}{4}$, then find the value of $(1+\tan A)(1+\tan B)$.

**OR**

**(b)** Find the value of $\sqrt{3}\ \text{cosec}\ 20^\circ - \sec 20^\circ$.

**23.** (i) Convert $250^\circ$ into radian.

(ii) Convert $3\pi$ radian into degree.

**24.** Find the values of the following:

(i) $\cos 210^\circ$ (ii) $\text{cosec}\ 390^\circ$

**25. (a)** If $\cot x = -\frac{5}{12}$ and $x$ lies in second quadrant, then find the values of other five trigonometric functions.

**OR**

**(b)** If $\sin\theta = \frac{12}{13}$ and $\theta$ lies in the second quadrant, then find the value of $\sec\theta + \tan\theta$.

## SECTION - C

**This section comprises of 6 Short Answer (SA) type questions of 3 marks each.** **6 × 3 = 18**

**26. (a)** If $\frac{2\sin\alpha}{1+\cos\alpha+\sin\alpha} = y$, then prove that $\frac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha}$ is also equal to $y$.

**OR**

**(b)** If $\sec\theta + \tan\theta = p$, obtain the values of $\sec\theta$, $\tan\theta$ and $\sin\theta$ in terms of $p$.

**27.** Prove that $\frac{\tan A+\sec A-1}{\tan A-\sec A+1}=\frac{1+\sin A}{\cos A}$.

**28. (a)** Prove that : $\sin x + \sin y + \sin z - \sin(x+y+z)$

$= 4\sin\frac{x+y}{2}\sin\frac{y+z}{2}\sin\frac{x+z}{2}$.

**OR**

**(b)** If $\tan\beta = \frac{n\sin\alpha\cos\alpha}{1-n\sin^2\alpha}$, then show that

$\tan(\alpha-\beta) = (1-n)\tan\alpha$.

**29.** Prove that :

$\cos x\cos\frac{x}{2} - \cos 3x\cos\frac{9x}{2} = \sin\frac{7x}{2}\sin 4x$.

**30. (a)** Prove that $\frac{\sin(x+y)}{\sin(x-y)} = \frac{\tan x+\tan y}{\tan x-\tan y}$.

**OR**

**(b)** Find the value of $\cos 570°\sin 510° + \sin(-330°)\cos(-390°)$.

**31.** Prove that $\tan\alpha\cdot\tan(60°-\alpha)\tan(60°+\alpha) = \tan 3\alpha$.

## SECTION - D

**This section comprises of 4 Long Answer (LA) type questions of 5 marks each.** **4 × 5 = 20**

**32.** If $\frac{\sin A}{\sin B} = p$ and $\frac{\cos A}{\cos B} = q$, then find $\tan A$ and $\tan B$.

**33. (a)** If $\frac{\sin^4\theta}{a} + \frac{\cos^4\theta}{b} = \frac{1}{a+b}$, then find the value of

$\frac{\sin^{12}\theta}{a^5} + \frac{\cos^{12}\theta}{b^5}$.

**OR**

**(b)** If $\frac{ax}{\cos\theta} + \frac{by}{\sin\theta} = a^2 - b^2$ and

$\frac{ax\sin\theta}{\cos^2\theta} - \frac{by\cos\theta}{\sin^2\theta} = 0$, then prove that

$(ax)^{2/3} + (by)^{2/3} = (a^2-b^2)^{2/3}$.

**34.** If $\sin(\theta+\alpha) = a$ and $\sin(\theta+\beta) = b$, then prove that $\cos 2(\alpha-\beta) - 4ab\cos(\alpha-\beta) = 1 - 2a^2 - 2b^2$.

**35. (a)** Find the value of

$\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{3\pi}{15}\cos\frac{4\pi}{15}\cos\frac{5\pi}{15}\cos\frac{6\pi}{15}\cos\frac{7\pi}{15}$.

$\left[\text{Given, } \sin 18° = \frac{\sqrt{5}-1}{4} \text{ and } \cos 36° = \frac{\sqrt{5}+1}{4}\right]$

**OR**

**(b)** Find the value of $\tan 6°\cdot\tan 42°\cdot\tan 66°\cdot\tan 78°$.

## SECTION - E

**This section comprises of 3 case-study/passage-based questions of 4 marks each.** **3 × 4 = 12**

**36.** Read the given passage and answer the following questions.

$\cos\frac{\pi}{7}$, $\cos\frac{3\pi}{7}$, $\cos\frac{5\pi}{7}$ are the roots of the equation $8x^3 - 4x^2 - 4x + 1 = 0$.

(i) Find the value of $\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}$.

(ii) Find the value of $\sec\frac{\pi}{7} + \sec\frac{3\pi}{7} + \sec\frac{5\pi}{7}$.

**37.** Amit and Purnima were discussing about trigonometric functions and identities. Amit said that I am an expert in solving problems based on trigonometry.

So, Purnima said that she will give him two values, $\sin A = \frac{-3}{5}$ and $\cos B = \frac{5}{13}$; $0 < B < \frac{\pi}{2}$ and $\pi < A < \frac{3\pi}{2}$ and asked to Amit to find out some other values.

Based on the given information, answer the following questions:

(i) What will be the value of $\cos A$ and $\sin B$?

(ii) Find the value of $\cos(A+B)$.

(iii) (a) Evaluate $\tan(A+B)$.

**OR**

(iii) (b) What will be the value of $\sin(A+B)\sin(A-B)$?

**38.** Sumit and Kavita were watching a clock, Kavita tells Sumit that every time the hour or minute hand of this clock moves, they subtend some angles at the centre.

Based on the given information, answer the following questions:

(i) If the minute hand of the clock is 18 cm long. How far does it moves in 10 minute?

(ii) Find the angle between the minute hand and the hour hand of the clock, when the time is 6:45 A.M.

(iii) (a) Find the length of an arc of a circle of radius 5 cm subtending a central angle measuring 15°. (Use $\pi = 3.14$)

**OR**

(iii) (b) Find the angle between the hour hand and the minute hand of the clock at half past two in radians.

## SOLUTIONS

**1. (b) :** We know that $180° = \pi$ radian

Hence, $40°20' = 40\frac{1}{3}$ degree $= \frac{121\pi}{540}$ radian

Therefore, $40°20' = \frac{121\pi}{540}$ radian.

**2. (b) :** Circumference of a circular wire of radius 7 cm $= 2\pi \times 7 = 14\pi$

As we know, $\theta = \frac{l}{r} \Rightarrow \theta = \frac{14\pi}{12} = \frac{7\pi \times 180°}{6\pi} = 210°$

**3. (b) :** Since, $180° = \pi$ radian

$\Rightarrow 1° = \frac{\pi}{180}$ radian $\Rightarrow 75° = \frac{75\pi}{180} = \frac{5\pi}{12}$ radian

**4. (a) :** We have, $\left(\frac{\pi}{9}\right)\text{radian} = \left(\frac{180}{\pi} \times \frac{\pi}{9}\right)^\circ = 20°$

$\left(-\frac{\pi}{3}\right)\text{radian} = \left(\frac{180}{\pi} \times \left(-\frac{\pi}{3}\right)\right)^\circ = -60°$

**5. (b) :** We have, $\frac{\sin x}{\cos x} \times \frac{\sec x}{\operatorname{cosec} x} \times \frac{\tan x}{\cot x} = 9$

$\Rightarrow \tan^4 x = 9$

$\Rightarrow \tan^2 x = 3 \Rightarrow \tan x = \pm\sqrt{3} \Rightarrow x = \frac{\pi}{3} \in \left(0, \frac{\pi}{2}\right)$

**6. (b) :** Given, $\sin A - \sqrt{6}\cos A = \sqrt{7}\cos A$ ...(i)

Let $\cos A + \sqrt{6}\sin A = x$ ...(ii)

Squaring (i) and (ii) and then adding, we get

$7 = 7\cos^2 A + x^2 \Rightarrow x = \sqrt{7}\sin A$

**7. (a) :** Given, $3\sin\theta + 5\cos\theta = 4$ ...(i)

Let $x = 5\sin\theta - 3\cos\theta$ ...(ii)

Squaring (i) and (ii) and then adding, we get

$9(\sin^2\theta + \cos^2\theta) + 25(\sin^2\theta + \cos^2\theta) = 16 + x^2$

$\Rightarrow x^2 = 18 \Rightarrow x = \pm\sqrt{18} = \pm 3\sqrt{2}$

**8. (c) :** We have, $x \sin 45° \cos^2 60° = \frac{\tan^2 60° \operatorname{cosec} 30°}{\sec 45° \cot^2 30°}$

$\Rightarrow x \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{4} = \frac{3 \cdot 2}{\sqrt{2} \cdot 3} \Rightarrow x = 8$

**9. (d) :** Since, $\sec^2 x = 1 + \tan^2 x = 1 + \left(\frac{3}{4}\right)^2 = \frac{25}{16}$

$\Rightarrow \sec x = \pm\frac{5}{4}$

But $x$ lies in third quadrant, so $\sec x = -\frac{5}{4}$

**10. (d) :** We have, $\sqrt{2}\sin\left(\frac{\pi}{4} - \theta\right)$

$= \sqrt{2}\left[\sin\frac{\pi}{4}\cos\theta - \cos\frac{\pi}{4}\sin\theta\right]$

$= \sqrt{2}\left[\frac{1}{\sqrt{2}}\cos\theta - \frac{1}{\sqrt{2}}\sin\theta\right] = \cos\theta - \sin\theta$

**11. (d) :** We have, $\cos 80° \cos 20° + \sin 80° \sin 20°$

$= \cos(80° - 20°) = \cos 60° = \frac{1}{2}$

**12. (b) :** We have, $\sin^2 17.5° + \sin^2 72.5°$

$= \sin^2 17.5° + \sin^2(90 - 17.5°)$

$= \sin^2 17.5° + \cos^2 17.5° = 1 = \tan^2 45°$

**13. (d) :** We have, $\sec\frac{2\pi}{3} + \operatorname{cosec}\frac{5\pi}{6}$

$= \sec\left(\pi - \frac{\pi}{3}\right) + \operatorname{cosec}\left(\pi - \frac{\pi}{6}\right)$

$= -\sec\frac{\pi}{3} + \operatorname{cosec}\frac{\pi}{6} = -2 + 2 = 0$

**14. (a) :** We have,

$\sin(40° + \theta)\cos(10° + \theta) - \cos(40° + \theta)\sin(10° + \theta)$

$= \sin\{(40° + \theta) - (10° + \theta)\} = \sin 30° = \frac{1}{2}$

**15. (b) :** $\frac{11}{15}\text{radian} = \left(\frac{180}{\pi} \times \frac{11}{15}\right)^\circ$

$= \left(\frac{180 \times 11 \times 7}{22 \times 15}\right)^\circ = 42°$

**16. (b) :** We have,

$\frac{\sin(90° + A)}{\cos(-A)} - \frac{\sin(180° - A)}{\sin(-A)} + \frac{\tan(270° + A)}{\cot(-A)}$

$= \frac{\cos A}{\cos A} - \frac{\sin A}{(-\sin A)} + \frac{(-\cot A)}{(-\cot A)} = 1 + 1 + 1 = 3$

**17. (c)**

**18. (d) :** We have, $4\sin A\cos^3 A - 4\cos A\sin^3 A$
$= 4\sin A\cos A(\cos^2 A - \sin^2 A) = 2\sin 2A\cos 2A = \sin 4A$

**19. (d) :** Let the radius of the circle be $r$ and angle of sector be $\theta$.

$\therefore$ Perimeter of sector $= 2r + r\theta$

Length of the arc of the semi-circle $= \pi r$

$\therefore \quad 2r + r\theta = \pi r \Rightarrow \theta = \pi - 2$

So, Assertion (A) is false and Reason (R) is a standard result, so its true.

**20. (a) :** $\because -1 \le \cos\theta \le 1 \Rightarrow -5 \le 5\cos\theta \le 5$

$\Rightarrow 12 - 5 \le 5\cos\theta + 12 \le 5 + 12 \Rightarrow 7 \le 5\cos\theta + 12 \le 17$

$\therefore$ Smallest value of $5\cos\theta + 12$ is 7.

So, Assertion (A) is true.

Reason (R) is a standard result.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

**21.** In 60 minutes, the minute hand of a watch completes one revolution.

In 40 minutes, the minute hand turns through 2/3 of a revolution.

Therefore, $\theta = (2/3)\times 360° = 240° = 240°\times\dfrac{\pi}{180°}$ radian
$= 4\pi/3$ radian.

Hence, the required distance travelled is given by

$l = r\theta = 1.5\times(4\pi/3) = 2\pi$ cm $= (2\times 3.14)$ cm $= 6.28$ cm

**22. (a)** We have, $A + B = 5\pi/4$

$$\Rightarrow \tan(A+B) = \tan\frac{5\pi}{4} = \tan\left(\pi+\frac{\pi}{4}\right) = \tan\frac{\pi}{4}$$

$$\Rightarrow \frac{\tan A+\tan B}{1-\tan A\tan B} = 1$$

$\Rightarrow \tan A + \tan B + \tan A\tan B = 1$

$\Rightarrow (1 + \tan A) + \tan B(1 + \tan A) = 1 + 1$

$\Rightarrow (1 + \tan A)(1 + \tan B) = 2$

**OR**

**(b)** We have, $\sqrt{3}$ cosec $20° - \sec 20°$

$$= \frac{\sqrt{3}}{\sin 20°} - \frac{1}{\cos 20°} = \frac{\sqrt{3}\cos 20° - \sin 20°}{\sin 20°\cos 20°}$$

$$= 4\left(\frac{\frac{\sqrt{3}}{2}\cos 20° - \frac{1}{2}\sin 20°}{2\sin 20°\cos 20°}\right)$$

$$= 4\left(\frac{\sin 60°\cos 20° - \cos 60°\sin 20°}{\sin 40°}\right)$$

$$= 4\left(\frac{\sin(60° - 20°)}{\sin 40°}\right) = 4$$

**23.** (i) To convert from degrees to radians, multiply degree measure by $\pi/180°$

So, $250° = \left(\dfrac{\pi}{180°}\times 250°\right)$ radian $= (25\pi/18)$ radian

(ii) To convert from radians to degrees, multiply radian measure by $180°/\pi$.

So, $3\pi$ radian $= \left(\dfrac{180°}{\pi}\times 3\pi\right)$ degree $= 540°$

**24.** (i) We have, $\cos 210° = \cos(180° + 30°)$

$[\because \cos(180° + \theta) = -\cos\theta]$

$= -\cos 30° = -\dfrac{\sqrt{3}}{2}$

(ii) We have, $\text{cosec}\, 390° = \text{cosec}(1\times 360° + 30°)$

$[\because \text{cosec}(n\times 360° + \theta) = \text{cosec}\,\theta,\ n\in Z]$

$= \text{cosec}\, 30° = 2$

**25. (a)** We have, $\cot x = -\dfrac{5}{12} \Rightarrow \tan x = -\dfrac{12}{5}$

Now, $\sec^2 x = 1 + \tan^2 x = 1 + \dfrac{144}{25} = \dfrac{169}{25}$

$\Rightarrow \quad \sec x = \pm\dfrac{13}{5}$

Since, $x$ lies in second quadrant, sec $x$ will be negative.

$\therefore \quad \sec x = -\dfrac{13}{5} \Rightarrow \cos x = -\dfrac{5}{13}$

Also, $\sin x = \tan x\times\cos x = \left(-\dfrac{12}{5}\right)\times\left(-\dfrac{5}{13}\right) = \dfrac{12}{13}$

$\Rightarrow \quad \text{cosec}\, x = \dfrac{13}{12}$

**OR**

**(b)** We know that $\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \quad \cos\theta = \pm\sqrt{1-\sin^2\theta}$

In the second quadrant, $\cos\theta$ is negative.

$\therefore \quad \cos\theta = -\sqrt{1-\sin^2\theta}$

Now, $\sec\theta + \tan\theta = \dfrac{1}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta} = \dfrac{1+\sin\theta}{\cos\theta}$

$$= \frac{1+\sin\theta}{-\sqrt{1-\sin^2\theta}} = \frac{1+\frac{12}{13}}{-\sqrt{1-\left(\frac{12}{13}\right)^2}} = \frac{\frac{25}{13}}{-\sqrt{\frac{25}{169}}} = \frac{\frac{25}{13}}{-\frac{5}{13}} = -5$$

**26. (a)** Given, $\frac{2\sin\alpha}{1+\cos\alpha+\sin\alpha} = y$ ... (i)

Consider, $\frac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha}$

$= \frac{\{(1+\sin\alpha)-\cos\alpha\}}{(1+\sin\alpha)} \cdot \frac{\{(1+\sin\alpha)+\cos\alpha\}}{\{(1+\sin\alpha)+\cos\alpha\}}$

$= \frac{(1+\sin\alpha)^2-\cos^2\alpha}{(1+\sin\alpha)(1+\sin\alpha+\cos\alpha)}$

$= \frac{(1+\sin^2\alpha+2\sin\alpha)-\cos^2\alpha}{(1+\sin\alpha)(1+\sin\alpha+\cos\alpha)}$

$= \frac{1+\sin^2\alpha+2\sin\alpha-1+\sin^2\alpha}{(1+\sin\alpha)(1+\sin\alpha+\cos\alpha)}$ $(\because \sin^2\alpha + \cos^2\alpha = 1)$

$= \frac{2\sin^2\alpha+2\sin\alpha}{(1+\sin\alpha)(1+\sin\alpha+\cos\alpha)}$

$= \frac{2\sin\alpha(1+\sin\alpha)}{(1+\sin\alpha)(1+\sin\alpha+\cos\alpha)}$

$= \frac{2\sin\alpha}{1+\sin\alpha+\cos\alpha} = y$ [From (i)]

Hence proved.

**OR**

**(b)** We have, $\sec\theta + \tan\theta = p$ ...(i)

We know that $\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

$\Rightarrow p(\sec\theta - \tan\theta) = 1$

$\Rightarrow \sec\theta - \tan\theta = \frac{1}{p}$ ...(ii)

Adding (i) and (ii), we get

$(\sec\theta + \tan\theta) + (\sec\theta - \tan\theta) = p + \frac{1}{p}$

$\Rightarrow 2\sec\theta = p + \frac{1}{p} \Rightarrow \sec\theta = \frac{p^2+1}{2p}$ ...(iii)

Subtracting (ii) from (i), we get

$(\sec\theta + \tan\theta) - (\sec\theta - \tan\theta) = p - \frac{1}{p}$

$\Rightarrow 2\tan\theta = p - \frac{1}{p}$

$\Rightarrow \tan\theta = \frac{p^2-1}{2p}$ ...(iv)

Dividing (iv) by (iii), we get

$\sin\theta = \frac{p^2-1}{p^2+1}$

**27.** Consider, L.H.S. $= \frac{\tan A+\sec A-1}{\tan A-\sec A+1}$

$= \frac{\tan A+\sec A-(\sec^2 A-\tan^2 A)}{(\tan A-\sec A+1)}$ $[\because \sec^2 A - \tan^2 A = 1]$

$= \frac{(\tan A+\sec A)-(\sec A+\tan A)(\sec A-\tan A)}{(1-\sec A+\tan A)}$

$= \frac{(\sec A+\tan A)(1-\sec A+\tan A)}{1-\sec A+\tan A}$

$= \sec A+\tan A = \frac{1}{\cos A}+\frac{\sin A}{\cos A} = \frac{1+\sin A}{\cos A}$ = R.H.S.

**28. (a)** Consider, L.H.S. = $(\sin x + \sin y) + (\sin z - \sin(x+y+z)) =$

$\left(2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)\right) + \left(2\cos\left(\frac{z+x+y+z}{2}\right)\sin\left(\frac{z-x-y-z}{2}\right)\right)$

$= 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) - 2\cos\left(\frac{x+y+2z}{2}\right)\sin\left(\frac{x+y}{2}\right)$

$= 2\sin\left(\frac{x+y}{2}\right)\left[\cos\left(\frac{x-y}{2}\right) - \cos\left(\frac{x+y+2z}{2}\right)\right]$

$= 2\sin\left(\frac{x+y}{2}\right)\left[\left(-2\sin\left(\frac{x-y+x+y+2z}{4}\right)\right)\sin\left(\frac{x-y-x-y-2z}{4}\right)\right]$

$= 2\sin\left(\frac{x+y}{2}\right)\left[2\sin\left(\frac{x+z}{2}\right)\sin\left(\frac{y+z}{2}\right)\right]$

$= 4\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{y+z}{2}\right)\sin\left(\frac{x+z}{2}\right)$ = R.H.S.

**OR**

**(b)** Consider, L.H.S. $= \tan(\alpha - \beta) = \frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$

$= \frac{\frac{\sin\alpha}{\cos\alpha} - \frac{n\sin\alpha\cos\alpha}{1-n\sin^2\alpha}}{1+\frac{\sin\alpha}{\cos\alpha}\times\frac{n\sin\alpha\cos\alpha}{1-n\sin^2\alpha}}$

$\left[\because \tan\beta = \frac{n\sin\alpha\cos\alpha}{1-n\sin^2\alpha}\text{[Given]}\right]$

$$= \frac{\sin\alpha - n\sin^3\alpha - n\sin\alpha\cos^2\alpha}{\cos\alpha(1-n\sin^2\alpha)+n\sin^2\alpha\cos\alpha}$$

$$= \frac{\sin\alpha - n\sin^3\alpha - n\sin\alpha(1-\sin^2\alpha)}{\cos\alpha - n\sin^2\alpha\cos\alpha + n\sin^2\alpha\cos\alpha}$$

$$= \frac{\sin\alpha - n\sin\alpha}{\cos\alpha} = \frac{\sin\alpha}{\cos\alpha}(1-n) = (1-n)\tan\alpha \ = \text{R.H.S.}$$

**29.** Consider, L.H.S. $= \cos x\cos\frac{x}{2} - \cos 3x\cos\frac{9x}{2}$

$$= \frac{1}{2}\left[2\cos x\cdot\cos\frac{x}{2} - 2\cos 3x\cdot\cos\frac{9x}{2}\right]$$

$$= \frac{1}{2}\left[\cos\left(x+\frac{x}{2}\right)+\cos\left(x-\frac{x}{2}\right) - \cos\left(3x+\frac{9x}{2}\right)-\cos\left(3x-\frac{9x}{2}\right)\right]$$

$[\because \cos(A+B)+\cos(A-B) = 2\cos A\cos B]$

$$= \frac{1}{2}\left[\cos\frac{3x}{2}+\cos\frac{x}{2}-\cos\frac{15x}{2}-\cos\frac{3x}{2}\right]$$

$$= \frac{1}{2}\left[\cos\frac{x}{2}-\cos\frac{15x}{2}\right]$$

$$= \frac{1}{2}\left[2\sin\left(\frac{x+15x}{4}\right)\sin\left(\frac{15x-x}{4}\right)\right]$$

$$\left[\because \cos A-\cos B = 2\sin\frac{A+B}{2}\sin\frac{B-A}{2}\right]$$

$= \sin 4x\cdot \sin\frac{7x}{2} = \text{R.H.S.}$

Hence proved.

**30. (a)** Consider, L.H.S. $= \frac{\sin(x+y)}{\sin(x-y)}$

$$= \frac{\sin x\cos y+\cos x\sin y}{\sin x\cos y-\cos x\sin y} \quad \ldots(i)$$

$[\because \sin(x+y) = \sin x\cos y+\cos x\sin y,$
$\sin(x-y) = \sin x\cos y-\cos x\sin y]$

Now, R.H.S. $= \frac{\tan x+\tan y}{\tan x-\tan y}$

$$= \frac{\frac{\sin x}{\cos x}+\frac{\sin y}{\cos y}}{\frac{\sin x}{\cos x}-\frac{\sin y}{\cos y}} = \frac{\frac{\sin x\cos y+\cos x\sin y}{\cos x\cos y}}{\frac{\sin x\cos y-\cos x\sin y}{\cos x\cos y}}$$

$$= \frac{\sin x\cos y+\cos x\sin y}{\sin x\cos y-\cos x\sin y} \quad \ldots(ii)$$

$\therefore$ From (i) and (ii), L.H.S. = R.H.S.

Hence Proved.

**OR**

**(b)** We have, $\cos 570°\sin 510° + \sin(-330°)\cos(-390°)$

$= \cos 570°\sin 510° - \sin 330°\cos 390°$

$[\because \sin(-x) = -\sin x$ and $\cos(-x) = \cos x]$

$= \cos(540°+30°)\sin(540°-30°) - \cos(360°+30°)\sin(360°-30°)$

$= -\cos 30°\sin 30° + \cos 30°\sin 30° = 0$

**31.** Consider, L.H.S. $= \tan\alpha\cdot\tan(60°-\alpha)\cdot\tan(60°+\alpha)$

$$= \frac{\sin\alpha}{\cos\alpha}\cdot\frac{\sin(60°-\alpha)}{\cos(60°-\alpha)}\cdot\frac{\sin(60°+\alpha)}{\cos(60°+\alpha)}$$

$$= \frac{\sin\alpha}{\cos\alpha}\cdot\frac{(\sin^2 60°-\sin^2\alpha)}{(\cos^2 60°-\sin^2\alpha)}$$

$[\because \sin(A+B)\sin(A-B) = \sin^2 A-\sin^2 B$
and $\cos(A+B)\cos(A-B) = \cos^2 A-\sin^2 B]$

$$= \frac{\sin\alpha\left(\frac{3}{4}-\sin^2\alpha\right)}{\cos\alpha\left(\frac{1}{4}-\sin^2\alpha\right)} = \frac{\sin\alpha(3-4\sin^2\alpha)}{\cos\alpha(1-4\sin^2\alpha)}$$

$$= \frac{3\sin\alpha-4\sin^3\alpha}{\cos\alpha[1-4(1-\cos^2\alpha)]} = \frac{3\sin\alpha-4\sin^3\alpha}{\cos\alpha(4\cos^2\alpha-3)}$$

$$= \frac{3\sin\alpha-4\sin^3\alpha}{4\cos^3\alpha-3\cos\alpha} = \frac{\sin 3\alpha}{\cos 3\alpha} = \tan 3\alpha \ = \text{R.H.S.}$$

Hence proved.

**32.** We have, $\frac{\sin A}{\sin B} = p$ ....(i) and $\frac{\cos A}{\cos B} = q$ ...(ii)

Dividing (i) by (ii), we get

$$\Rightarrow \frac{\sin A}{\sin B}\cdot\frac{\cos B}{\cos A} = \frac{p}{q}$$

$$\Rightarrow \frac{\tan A}{\tan B} = \frac{p}{q} \Rightarrow \frac{\tan A}{p} = \frac{\tan B}{q} = \lambda(\text{say})$$

$\Rightarrow \tan A = p\lambda$ and $\tan B = q\lambda$ ...(iii)

Now, $\sin A = p\sin B$

$$\Rightarrow \frac{\tan A}{\sqrt{1+\tan^2 A}} = p\frac{\tan B}{\sqrt{1+\tan^2 B}}$$

$$\left[\because \sin\theta = \frac{\tan\theta}{\sqrt{1+\tan^2\theta}}\right]$$

$$\Rightarrow \frac{p\lambda}{\sqrt{1+p^2\lambda^2}} = p\frac{q\lambda}{\sqrt{1+q^2\lambda^2}}$$

$\Rightarrow 1+q^2\lambda^2 = q^2(1+p^2\lambda^2) \Rightarrow \lambda^2(q^2-p^2q^2) = q^2-1$

$$\Rightarrow \lambda^2 = \frac{q^2-1}{q^2(1-p^2)} \Rightarrow \lambda = \pm\frac{1}{q}\sqrt{\frac{q^2-1}{1-p^2}}$$

$$\therefore \quad \tan A = \pm\frac{p}{q}\sqrt{\frac{q^2-1}{1-p^2}} \text{ and } \tan B = \pm\sqrt{\frac{q^2-1}{1-p^2}}$$

**33. (a)** Given, $\frac{\sin^4\theta}{a}+\frac{\cos^4\theta}{b}=\frac{1}{a+b}$

$$\Rightarrow \frac{(\sin^2\theta)^2}{a}+\frac{(1-\sin^2\theta)^2}{b}=\frac{1}{a+b}$$

$\Rightarrow \lambda^2 b(a+b) + (a+b)\,a\,(1-\lambda)^2 = ab$, where $\lambda = \sin^2\theta$

$\Rightarrow \lambda^2 ab + \lambda^2 b^2 + a^2 + a^2\lambda^2 - 2a^2\lambda + ab + ab\lambda^2 - 2ab\lambda = ab$

$\Rightarrow \lambda^2\{a^2 + b^2 + 2ab\} + a^2 - 2\lambda a(a+b) = 0$

$\Rightarrow \lambda^2(a+b)^2 + a^2 - 2\lambda a\,(a+b) = 0 \Rightarrow \{\lambda\,(a+b) - a\}^2 = 0$

$$\Rightarrow \lambda = \frac{a}{a+b} \Rightarrow \sin^2\theta = \frac{a}{a+b} \qquad \text{...(i)}$$

From (i), $\cos^2\theta = 1 - \sin^2\theta = 1 - \frac{a}{a+b} = \frac{b}{a+b}$ ...(ii)

Now, $\frac{\sin^{12}\theta}{a^5}+\frac{\cos^{12}\theta}{b^5}=\frac{(\sin^2\theta)^6}{a^5}+\frac{(\cos^2\theta)^6}{b^5}$

$$= \frac{\left(\frac{a}{a+b}\right)^6}{a^5}+\frac{\left(\frac{b}{a+b}\right)^6}{b^5} \qquad [\because \text{ Using (i) and (ii)}]$$

$$= \frac{a+b}{(a+b)^6} = \frac{1}{(a+b)^5}$$

**OR**

**(b)** We have, $\frac{ax}{\cos\theta}+\frac{by}{\sin\theta}=a^2-b^2$ ...(i)

and $\frac{ax\sin\theta}{\cos^2\theta}-\frac{by\cos\theta}{\sin^2\theta}=0 \Rightarrow by=\frac{ax\sin^3\theta}{\cos^3\theta}$ ...(ii)

Substituting the value of $by$ in (i), we get

$$\frac{ax}{\cos\theta}+\frac{ax\sin^2\theta}{\cos^3\theta}=a^2-b^2$$

$\Rightarrow ax\,(\cos^2\theta + \sin^2\theta) = (a^2 - b^2)\cos^3\theta$

$\Rightarrow ax = (a^2 - b^2)\cos^3\theta$ ...(iii)

Substituting the value of $ax$ in (ii), we get

$by = (a^2 - b^2)\sin^3\theta$ ...(iv)

$\therefore (ax)^{2/3} + (by)^{2/3} = ((a^2-b^2)\cos^3\theta)^{2/3} + ((a^2-b^2)\sin^3\theta)^{2/3}$

(By using (iii) and (iv))

$= (a^2 - b^2)^{2/3}(\cos^2\theta + \sin^2\theta) = (a^2 - b^2)^{2/3}$

Hence proved.

**34.** Given, $\sin(\theta + \alpha) = a$ ...(i)

and $\sin(\theta + \beta) = b$ ...(ii)

$\therefore \quad \cos(\theta+\alpha) = \sqrt{1-a^2}$ and $\cos(\theta+\beta) = \sqrt{1-b^2}$

Since, $\cos(\alpha - \beta) = \cos[(\theta + \alpha) - (\theta + \beta)]$

$= \cos(\theta + \alpha)\cos(\theta + \beta) + \sin(\theta + \alpha)\sin(\theta + \beta)$

$[\because \quad \cos(A - B) = \cos A\cos B + \sin A\sin B]$

$$= \sqrt{1-a^2}\sqrt{1-b^2} + ab = ab + \sqrt{(1-a^2)(1-b^2)}$$

$$\Rightarrow \cos(\alpha-\beta) = ab + \sqrt{1-a^2-b^2+a^2b^2}$$

Now, L.H.S. $= \cos2(\alpha - \beta) - 4ab\cos(\alpha - \beta)$

$= 2\cos^2(\alpha - \beta) - 1 - 4ab\cos(\alpha - \beta)$

$$= 2\left[ab + \sqrt{1-a^2-b^2+a^2b^2}\right]^2 - 1 - 4ab\left[ab + \sqrt{1-a^2-b^2+a^2b^2}\right]$$

$$= 2\left[(ab)^2 + 1 - a^2 - b^2 + a^2b^2 + 2ab\sqrt{1-a^2-b^2+a^2b^2}\right] - 1 - 4a^2b^2 - 4ab\sqrt{1-a^2-b^2+a^2b^2}$$

$$= 2a^2b^2 + 2 - 2a^2 - 2b^2 + 2a^2b^2 + 4ab\sqrt{1-a^2-b^2+a^2b^2} - 1 - 4a^2b^2 - 4ab\sqrt{1-a^2-b^2+a^2b^2}$$

$= 1 - 2a^2 - 2b^2 =$ R.H.S.

**35. (a)** We have,

$$\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{3\pi}{15}\cos\frac{4\pi}{15}\cos\frac{5\pi}{15}\cos\frac{6\pi}{15}\cos\frac{7\pi}{15}$$

$$= \left\{\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{7\pi}{15}\right\}\left\{\cos\frac{3\pi}{15}\cos\frac{6\pi}{15}\right\}\cos\frac{5\pi}{15}$$

$$= \frac{1}{2}\left\{\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{7\pi}{15}\right\}\left\{\cos\frac{3\pi}{15}\cos\frac{6\pi}{15}\right\}$$

$$= -\frac{1}{2}\left\{\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\right\}\left\{\cos\frac{3\pi}{15}\cos\frac{6\pi}{15}\right\}$$

$$\left[\because \cos\frac{7\pi}{15} = \cos\left(\pi - \frac{8\pi}{15}\right) = -\cos\frac{8\pi}{15}\right]$$

$$= \frac{-1}{2}\left\{\frac{2\sin\frac{\pi}{15}\cos\frac{\pi}{15}}{2\sin\frac{\pi}{15}}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\right\}\left\{\cos\frac{3\pi}{15}\cos\frac{6\pi}{15}\right\}$$

$$= \frac{-1}{2}\left\{\frac{2\sin\frac{2\pi}{15}\cos\frac{2\pi}{15}}{2^2\sin\frac{\pi}{15}}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\right\}\left\{\cos\frac{3\pi}{15}\cos\frac{6\pi}{15}\right\}$$

$$=-\frac{1}{2}\left\{\frac{\sin 2^4\frac{\pi}{15}}{2^4\sin\frac{\pi}{15}}\right\}\times\left\{\frac{\sin\left(2^2\times\frac{3\pi}{15}\right)}{2^2\sin\frac{3\pi}{15}}\right\}$$

$$=-\frac{1}{2}\left\{\frac{\sin\frac{16\pi}{15}}{16\sin\frac{\pi}{15}}\right\}\times\left\{\frac{\left(\sin\frac{12\pi}{15}\right)}{4\sin\frac{3\pi}{15}}\right\}$$

$$=-\frac{1}{2}\left\{\frac{\sin\left(\pi+\frac{\pi}{15}\right)}{16\sin\frac{\pi}{15}}\right\}\times\left\{\frac{\sin\left(\pi-\frac{3\pi}{15}\right)}{4\sin\frac{3\pi}{15}}\right\}$$

$$=-\frac{1}{2}\left\{\frac{-\sin\frac{\pi}{15}}{16\sin\frac{\pi}{15}}\right\}\times\left\{\frac{\sin\frac{3\pi}{15}}{4\sin\frac{3\pi}{15}}\right\}$$

$$=\left(-\frac{1}{2}\right)\times\left(-\frac{1}{16}\right)\times\frac{1}{4}=\frac{1}{128}$$

**OR**

**(b)** We have, $\tan6^\circ\cdot\tan42^\circ\cdot\tan66^\circ\cdot\tan78^\circ$

$$=\frac{\sin6^\circ}{\cos6^\circ}\cdot\frac{\sin42^\circ}{\cos42^\circ}\cdot\frac{\sin66^\circ}{\cos66^\circ}\cdot\frac{\sin78^\circ}{\cos78^\circ}$$

$$=\frac{2\sin66^\circ\sin6^\circ}{2\cos66^\circ\cos6^\circ}\cdot\frac{2\sin78^\circ\sin42^\circ}{2\cos78^\circ\cos42^\circ}$$

$$=\frac{\cos(66^\circ-6^\circ)-\cos(66^\circ+6^\circ)}{\cos(66^\circ+6^\circ)+\cos(66^\circ-6^\circ)}$$

$$\cdot\frac{\cos(78^\circ-42^\circ)-\cos(78^\circ+42^\circ)}{\cos(78^\circ+42^\circ)+\cos(78^\circ-42^\circ)}$$

$$=\frac{\cos60^\circ-\cos72^\circ}{\cos60^\circ+\cos72^\circ}\cdot\frac{\cos36^\circ-\cos120^\circ}{\cos36^\circ+\cos120^\circ}$$

$$=\left(\frac{\cos60^\circ-\cos(90^\circ-18^\circ)}{\cos60^\circ+\cos(90^\circ-18^\circ)}\right)\cdot\left(\frac{\cos36^\circ-\cos(90^\circ+30^\circ)}{\cos36^\circ+\cos(90^\circ+30^\circ)}\right)$$

$$=\left(\frac{\cos60^\circ-\sin18^\circ}{\cos60^\circ+\sin18^\circ}\right)\cdot\left(\frac{\cos36^\circ+\sin30^\circ}{\cos36^\circ-\sin30^\circ}\right)$$

$$=\left(\frac{\frac{1}{2}-\frac{\sqrt{5}-1}{4}}{\frac{1}{2}+\frac{\sqrt{5}-1}{4}}\right)\cdot\left(\frac{\frac{\sqrt{5}+1}{4}+\frac{1}{2}}{\frac{\sqrt{5}+1}{4}-\frac{1}{2}}\right)$$

$$=\frac{3-\sqrt{5}}{1+\sqrt{5}}\times\frac{3+\sqrt{5}}{\sqrt{5}-1}=\frac{9-5}{5-1}=\frac{4}{4}=1$$

[Given]

**36.** (i) Since, $\cos\frac{\pi}{7}, \cos\frac{3\pi}{7}, \cos\frac{5\pi}{7}$ are the roots of the equation $8x^3 - 4x^2 - 4x + 1 = 0$.

$\therefore$ Product of roots $=\cos\frac{\pi}{7}\cdot\cos\frac{3\pi}{7}\cdot\cos\frac{5\pi}{7}=-\frac{1}{8}$ ...(i)

Now, $\sin\frac{\pi}{14}\cdot\sin\frac{3\pi}{14}\cdot\sin\frac{5\pi}{14}$

$$=\sin\left(\frac{\pi}{2}-\frac{3\pi}{7}\right)\cdot\sin\left(\frac{\pi}{2}-\frac{2\pi}{7}\right)\cdot\sin\left(\frac{\pi}{2}-\frac{\pi}{7}\right)$$

$$=\cos\frac{3\pi}{7}\cdot\cos\frac{2\pi}{7}\cdot\cos\frac{\pi}{7}=-\cos\frac{\pi}{7}\cdot\cos\frac{3\pi}{7}\cdot\cos\left(\pi-\frac{2\pi}{7}\right)$$

$$=-\left[\cos\frac{\pi}{7}\cdot\cos\frac{3\pi}{7}\cdot\cos\frac{5\pi}{7}\right]=-\left(\frac{-1}{8}\right)=\frac{1}{8}$$ [Using (i)]

(ii) We have, $8x^3 - 4x^2 - 4x + 1 = 0$ ...(i)

Replacing $x$ by $\frac{1}{x}$, we get $\frac{8}{x^3}-\frac{4}{x^2}-\frac{4}{x}+1=0$

$\Rightarrow\ x^3 - 4x^2 - 4x + 8 = 0$ ...(ii)

Since, $\cos\frac{\pi}{7}$, $\cos\frac{3\pi}{7}$, $\cos\frac{5\pi}{7}$ are the roots of (i).

# SAMURAI SUDOKU

ANSWER - JUNE 2025

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 3 | 9 | 8 | 2 | 4 | 7 | 1 | 5 | 6 |
| 6 | 2 | 5 | 3 | 8 | 1 | 7 | 9 | 4 |
| 4 | 1 | 7 | 5 | 6 | 9 | 8 | 3 | 2 |
| 7 | 6 | 9 | 1 | 3 | 2 | 5 | 4 | 8 |
| 5 | 8 | 1 | 4 | 7 | 6 | 9 | 2 | 3 |
| 2 | 4 | 3 | 8 | 9 | 5 | 1 | 1 | 7 |
| 1 | 3 | 4 | 6 | 5 | 8 | 2 | 7 | 9 |
| 9 | 5 | 6 | 7 | 2 | 3 | 4 | 8 | 1 |
| 8 | 7 | 2 | 9 | 1 | 4 | 3 | 6 | 5 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 9 | 4 | 6 | 2 | 8 | 7 | 5 | 1 | 3 |
| 3 | 5 | 1 | 9 | 4 | 6 | 8 | 2 | 7 |
| 8 | 7 | 2 | 1 | 3 | 5 | 9 | 4 | 6 |
| 1 | 6 | 8 | 3 | 2 | 4 | 7 | 5 | 9 |
| 5 | 2 | 4 | 6 | 7 | 9 | 1 | 3 | 8 |
| 7 | 9 | 3 | 5 | 1 | 8 | 4 | 6 | 2 |
| 4 | 8 | 5 | 7 | 6 | 2 | 3 | 9 | 1 |
| 6 | 3 | 9 | 8 | 5 | 1 | 2 | 7 | 4 |
| 2 | 1 | 7 | 4 | 9 | 3 | 6 | 8 | 5 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | 6 | 1 | 4 |
| | | | | | | 8 | 3 | 2 |
| | | | | | | 5 | 9 | 7 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 8 | 2 | 5 | 9 | 7 | 3 | | | |
| 6 | 7 | 9 | 1 | 5 | 4 | | | |
| 3 | 1 | 4 | 8 | 2 | 6 | | | |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 9 | 3 | 7 | 2 | 5 | 6 | 1 | 4 | 8 |
| 2 | 4 | 1 | 3 | 7 | 8 | 9 | 5 | 6 |
| 5 | 6 | 8 | 1 | 4 | 9 | 7 | 2 | 3 |
| 3 | 1 | 5 | 8 | 6 | 2 | 4 | 9 | 7 |
| 4 | 9 | 6 | 7 | 3 | 1 | 2 | 8 | 5 |
| 8 | 7 | 2 | 5 | 9 | 4 | 6 | 3 | 1 |
| 6 | 8 | 4 | 9 | 1 | 5 | 3 | 7 | 2 |
| 1 | 5 | 3 | 4 | 2 | 7 | 8 | 6 | 9 |
| 7 | 2 | 9 | 6 | 8 | 3 | 5 | 1 | 4 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 5 | 6 | 7 | 3 | 9 | 2 | 6 | 7 | 1 |
| 2 | 8 | 3 | 7 | 4 | 1 | 3 | 5 | 8 |
| 4 | 9 | 1 | 5 | 6 | 8 | 9 | 2 | 4 |
| | | | 1 | 8 | 7 | 5 | 4 | 3 |
| | | | 4 | 3 | 5 | 2 | 9 | 6 |
| | | | 9 | 2 | 6 | 1 | 8 | 7 |
| | | | 6 | 5 | 9 | 7 | 1 | 2 |
| | | | 2 | 1 | 4 | 8 | 3 | 5 |
| | | | 8 | 7 | 3 | 4 | 6 | 9 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 5 | 4 | 8 | | | | | | |
| 8 | 2 | 9 | 6 | | | | | |
| 7 | 1 | 3 | | | | | | |
| 3 | 6 | 2 | 9 | | | | | |
| 1 | 8 | 7 | 4 | | | | | |
| 4 | 3 | 5 | 2 | | | | | |
| 9 | 2 | 6 | 1 | 8 | 7 | 4 | 3 | 5 |
| 6 | 5 | 9 | 7 | 1 | 2 | 3 | 8 | 4 |
| 2 | 1 | 4 | 8 | 3 | 5 | 9 | 6 | 7 |
| 8 | 7 | 3 | 4 | 6 | 9 | 1 | 5 | 2 |

$\therefore \quad \frac{1}{\cos\frac{\pi}{7}}, \frac{1}{\cos\frac{3\pi}{7}}, \frac{1}{\cos\frac{5\pi}{7}}$ are the roots of (ii)

$\Rightarrow \sec\frac{\pi}{7}, \sec\frac{3\pi}{7}, \sec\frac{5\pi}{7}$ are the roots of (ii)

Now, sum of roots $= \sec\frac{\pi}{7} + \sec\frac{3\pi}{7} + \sec\frac{5\pi}{7} = -\frac{(-4)}{1} = 4$

**37.** (i) Since, $\cos^2 A = 1 - \sin^2 A = 1 - \frac{9}{25} = \frac{16}{25}$

$\Rightarrow \cos A = \frac{-4}{5}$ as $\pi < A < \frac{3\pi}{2}$

Also, $\sin^2 B = 1 - \cos^2 B = 1 - \frac{25}{169} = \frac{144}{169}$

$\Rightarrow \sin B = \frac{12}{13}$ as $0 < B < \frac{\pi}{2}$

(ii) Consider, $\cos(A + B) = \cos A \cos B - \sin A \sin B$

$= \frac{-4}{5} \times \frac{5}{13} - \left(\frac{-3}{5}\right)\left(\frac{12}{13}\right) = \frac{-20}{65} + \frac{36}{65} = \frac{16}{65}$

(iii) (a) $\tan A = \frac{\sin A}{\cos A} = \frac{-3/5}{-4/5} = \frac{3}{4}$

Also, $\tan B = \frac{12/13}{5/13} = \frac{12}{5}$

Now, $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{\frac{3}{4} + \frac{12}{5}}{1 - \frac{3}{4} \times \frac{12}{5}} = \frac{-63}{16}$

So, $\tan(A + B) = \frac{-63}{16}$

**OR**

(iii) (b) We know that $\sin(A + B) \sin(A - B)$

$= \sin^2 A - \sin^2 B$

$= \left(\frac{-3}{5}\right)^2 - \left(\frac{12}{13}\right)^2 = \frac{9}{25} - \frac{144}{169} = \frac{1521 - 3600}{4225} = \frac{-2079}{4225}$

**38.** (i) Angle traced by the minute hand in 10 minutes

$= (10 \times 6)^\circ = 60^\circ = \left(60^\circ \times \frac{\pi}{180^\circ}\right)$ radian $= \frac{\pi}{3}$ radian

Now, $\theta = \frac{\text{arc length}}{\text{radius}}$

$\Rightarrow$ Arc length $= \left(18 \times \frac{\pi}{3}\right)$ cm $= \left(\frac{18}{3} \times \frac{22}{7}\right)$ cm $= \frac{132}{7}$ cm

$= 18.86$ cm (approx)

$\therefore$ Distance covered by minute hand in 10 minutes is 18.86 cm.

(ii) Angle traced by the hour hand in 12 hours $= 360^\circ$

Also, angle traced by the hour hand in 6 hours 45 min.

$= \frac{27}{4}$ hours $= \left(\frac{360}{12} \times \frac{27}{4}\right)^\circ = \left(\frac{405}{2}\right)^\circ = (202.5)^\circ$

and angle traced by minute hand in 45 minutes

$= \left(\frac{360}{60} \times 45\right)^\circ = 270^\circ$

$\therefore$ Required angle $= 270^\circ - (202.5)^\circ = \left(67\frac{1}{2}\right)^\circ$

(iii) (a) Let $\theta$ be the angle subtended by an arc of length $l$ at the centre of circle having radius 5 cm.

Now, $\theta = 15^\circ = \left(15 \times \frac{\pi}{180}\right)$ radian

We know that $l = r\theta$

$= 5 \times 15 \times \frac{\pi}{180} = \frac{5\pi}{12} = \frac{5 \times 3.14}{12} = 1.308$ cm.

**OR**

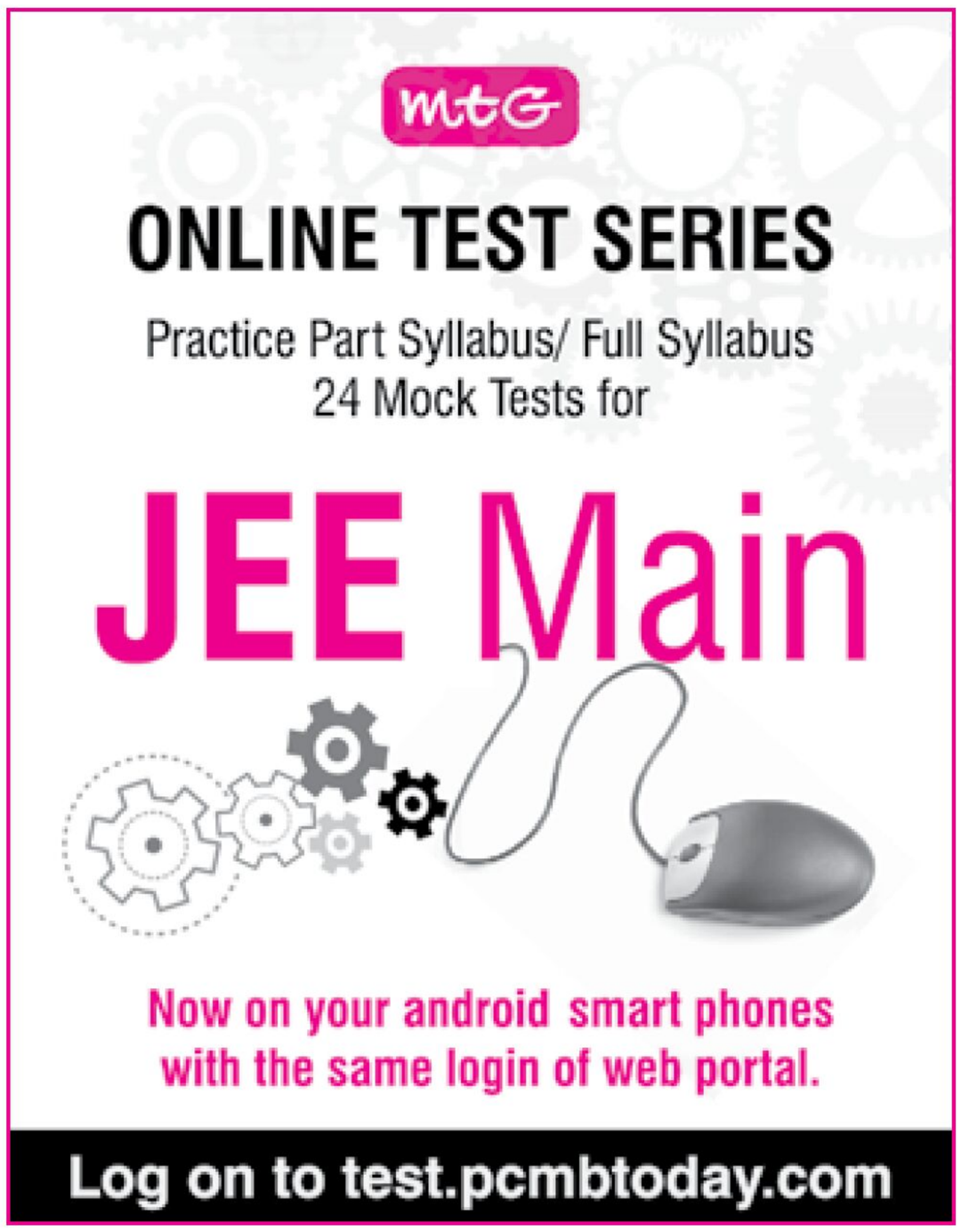
(iii) (b) The angle traced by the hour hand in 2 hours 30 min. *i.e.*, in $\frac{5}{2}$ hours $= \left(\frac{360}{12} \times \frac{5}{2}\right)^\circ = 75^\circ$

The angle traced by minute hand in 30 min.

$= \left(\frac{360}{60} \times 30\right)^\circ = 180^\circ$

Hence, required angle $= 180^\circ - 75^\circ = 105^\circ$

$= \left(\frac{\pi}{180} \times 105\right)$ radian $= \frac{7\pi}{12}$ radian ◈◈

Class XI

# Monthly test drive

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

## Series-2 Trigonometric Functions, Complex Numbers and Quadratic Equations

**Total Marks :** 80 **Time taken :** 60 Min.

### Only One Option Correct Type

**1.** $96\cos\frac{\pi}{33}\cos\frac{2\pi}{33}\cos\frac{4\pi}{33}\cos\frac{8\pi}{33}\cos\frac{16\pi}{33}$ is equal to

(a) 3 (b) 2 (c) 1 (d) 4

**2.** Two circles $C_1$ and $C_2$ have radii 18 and 12 units, respectively. If an arc of length $l$ of $C_1$ subtends an angle 80° at the centre, then the angle subtended by an arc of same length $l$ of $C_2$ at the centre is

(a) 90° (b) 100° (c) 110° (d) 120°

**3.** If $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$, then the value of $\cos 4x$ is

(a) $\frac{1}{3}$ (b) $\frac{2}{9}$ (c) $-\frac{7}{9}$ (d) $-\frac{3}{5}$

**4.** If $z = x - iy$ and $z^{1/3} = p + iq$, then $\frac{1}{p^2+q^2}\left(\frac{x}{p}+\frac{y}{q}\right)$ is equal to

(a) −2 (b) −1 (c) 1 (d) 2

**5.** If the center and radius of the circle $\left|\frac{z-2}{z-3}\right| = 2$ are respectively $(\alpha, \beta)$ and $\gamma$, then $3(\alpha+\beta+\gamma)$ is equal to

(a) 12 (b) 11 (c) 10 (d) 9

**6.** It is given that $z_r = \cos(\pi/2^r) + i\sin(\pi/2^r)$. The value of the product $z_1 \cdot z_2 \cdot z_3 \cdot z_4 \cdot \ldots\ldots\infty$ is

(a) 0 (b) 1 (c) −1 (d) $1+i$

### One or More Than One Option(s) Correct Type

**7.** Let $\alpha$ and $\beta$ be non-zero real numbers such that $2(\cos\beta - \cos\alpha) + \cos\alpha\cos\beta = 1$. Then which of the following is/are true?

(a) $\tan\left(\frac{\alpha}{2}\right) + \sqrt{3}\tan\left(\frac{\beta}{2}\right) = 0$

(b) $\tan\left(\frac{\alpha}{2}\right) - \sqrt{3}\tan\left(\frac{\beta}{2}\right) = 0$

(c) $\sqrt{3}\tan\left(\frac{\alpha}{2}\right) + \tan\left(\frac{\beta}{2}\right) = 0$

(d) $\sqrt{3}\tan\left(\frac{\alpha}{2}\right) - \tan\left(\frac{\beta}{2}\right) = 0$

**8.** The complex number $z$ satisfying the equation $|z-i| = |z+1| = 1$ is

(a) 0 (b) $1+i$

(c) $-1+i$ (d) $1-i$

**9.** Choose the correct statement :

(a) $x + \sin 2x$ is a periodic function

(b) $x + \sin 2x$ is not a periodic function

(c) $\cos\left(\sqrt{x}+1\right)$ is a periodic function

(d) $\cos\left(\sqrt{x}+1\right)$ is not a periodic function

**10.** Let $a$, $b$, $x$ and $y$ be real numbers such that $a - b = 1$ and $y \neq 0$. If the complex number $z = x + iy$ satisfies $\text{Im}\left(\frac{az+b}{z+1}\right) = y$, then which of the following is (are) possible value (s) of $x$?

(a) $1+\sqrt{1+y^2}$ (b) $-1+\sqrt{1-y^2}$

(c) $1-\sqrt{1+y^2}$ (d) $-1-\sqrt{1-y^2}$

**11.** If $|z+i|-|z-1|=|z|-2=0$ for a complex number $z$, then $z=$

(a) $\sqrt{2}(1+i)$ (b) $\sqrt{2}(1-i)$

(c) $\sqrt{2}(-1+i)$ (d) $\sqrt{2}(-1-i)$

**12.** Let $f(x)=x^2+x\sin x-\cos x$. Then

(a) $f(x)=0$ has at least one real root

(b) $f(x)=0$ has no real root

(c) $f(x)=0$ has one positive root

(d) $f(x)=0$ has one negative root

**13.** If the equation $x^2-cx+d=0$ has roots equal to the fourth powers of the roots of $x^2+ax+b=0$, where $a_2>4b$, then the roots of $x^2-4bx+2b^2-c=0$ will be

(a) both real

(b) both negative

(c) both positive

(d) one positive and one negative

## Comprehension Type

### Paragraph for Q. No. 14 and 15

Let $p$, $q$ be integers and let $\alpha$, $\beta$ be the roots of the equation, $x^2-x-1=0$, where $\alpha\neq\beta$. For $n=0,1,2,....$, let $a_n=p\alpha^n+q\beta^n$.

**FACT** : If $a$ and $b$ are rational numbers and $a+b\sqrt{5}=0$, then $a=0=b$.

**14.** $a_{12}$ is equal to

(a) $2a_{11}+a_{10}$ (b) $a_{11}-a_{10}$

(c) $a_{11}+2a_{10}$ (d) $a_{11}+a_{10}$

**15.** If $a_4=28$, then the value of $p+2q$ is

(a) 21 (b) 14 (c) 7 (d) 12

## Matrix Match Type

**16.** Match the following.

| Column-I | | Column-II | |
|---|---|---|---|
| (P) | $\left\{x\in\left[-\frac{2\pi}{3},\frac{2\pi}{3}\right]:\cos x+\sin x=1\right\}$ | (1) | has two elements |
| (Q) | $\left\{x\in\left[-\frac{5\pi}{18},\frac{5\pi}{18}\right]:\sqrt{3}\tan 3x=1\right\}$ | (2) | has three elements |
| (R) | $\left\{x\in\left[-\frac{6\pi}{5},\frac{6\pi}{5}\right]:2\cos(2x)=\sqrt{3}\right\}$ | (3) | has four elements |
| (S) | $\left\{x\in\left[-\frac{7\pi}{4},\frac{7\pi}{4}\right]:\sin x-\cos x=1\right\}$ | (4) | has five elements |
| | | (5) | has six elements |

The correct option is :

(a) (P) $\to$ (1); (Q) $\to$ (4); (R) $\to$ (1); (S) $\to$ (4)

(b) (P) $\to$ (1); (Q) $\to$ (1); (R) $\to$ (5); (S) $\to$ (3)

(c) (P) $\to$ (2); (Q) $\to$ (1); (R) $\to$ (5); (S) $\to$ (4)

(d) (P) $\to$ (2); (Q) $\to$ (4); (R) $\to$ (1); (S) $\to$ (3)

## Numerical Answer Type

**17.** If $\sqrt{3}(\cos^2 x)=(\sqrt{3}-1)\cos x+1$, the number of solutions of the given equation when $x\in\left[0,\frac{\pi}{2}\right]$ is _____.

**18.** The number of real solutions of the equation $(\sin x-x)(\cos x-x^2)=0$ is _____.

**19.** Let $\alpha=8-14i$, $A=\left\{z\in\mathbb{C}:\frac{\alpha z-\bar{\alpha}\,\bar{z}}{z^2-(\bar{z})^2-112i}=1\right\}$ and $B=\{z\in\mathbb{C}:|z+3i|=4\}$.

Then $\sum_{z\in A\cap B}(\mathrm{Re}\,z-\mathrm{Im}\,z)$ is equal to _____.

**20.** If the angle of elevation of a cloud from a point $P$ which is 25 m above a lake be 30° and the angle of depression of reflection of the cloud in the lake from $P$ be 60°, then the height of the cloud (in metres) from the surface of the lake is _____.

*Keys are published in this issue. Search now!* ☺

**SELF CHECK**

No. of questions attempted .....

No. of questions correct .....

Marks scored in percentage .....

**Check your score! If your score is**

| | | |
|---|---|---|
| > 90% | EXCELLENT WORK ! | You are well prepared to take the challenge of final exam. |
| 90-75% | GOOD WORK ! | You can score good in the final exam. |
| 74-60% | SATISFACTORY ! | You need to score more next time. |
| < 60% | NOT SATISFACTORY! | Revise thoroughly and strengthen your concepts. |

# CBSE

# Warm-up!

CLASS-XII

*Chapterwise practice questions for CBSE Exams as per the latest pattern and syllabus by CBSE for the academic session 2025-26.*

## Series-3 Matrices & Determinants

***General Instructions : Read the following instructions very carefully and strictly follow them :***

(i) This question paper contains **38** questions. **All** questions are **compulsory**.

(ii) This question paper is divided into **five** Sections – **A, B, C, D** and **E**.

(iii) In **Section A**, Questions no. **1** to **18** are multiple choice questions (MCQs) and Questions no. **19 and 20** are Assertion-Reason based questions of **1** mark each.

(iv) In **Section B**, Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.

(v) In **Section C**, Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.

(vi) In **Section D**, Questions no. **32** to **35** are long answer (LA) type questions, carrying **5** marks each.

(vii) In **Section E**, Questions no. **36** to **38** are case study based questions, carrying **4** marks each.

(viii) There is no overall choice. However, an internal choice has been provided in **2** questions in Section **B**, **3** questions in Section **C**, **2** questions in Section **D** and **2** questions in Section E.

(ix) Use of calculators is **NOT** allowed.

**Time Allowed :** 3 hours **Maximum Marks :** 80

### SECTION - A

**This section comprises of 20 Multiple Choice Questions (MCQs) of 1 mark each.** **20 × 1 = 20**

**1.** If $A=\begin{bmatrix} 3 & x-1 \\ 2x+3 & x+2 \end{bmatrix}$ is a symmetric matrix, then $x$ equal to

(a) 4 (b) 3 (c) –4 (d) –3

**2.** If $A=\begin{bmatrix} x & x-1 \\ 2x & 1 \end{bmatrix}$ and det $A = -9$, then the values of $x$ are

(a) $\frac{3}{2}, -3$ (b) $\frac{-2}{3}, 3$

(c) $\frac{2}{3}, 3$ (d) $\frac{-3}{2}, 3$

**3.** If $A^2 - A + I = O$, then the inverse of $A$ is

(a) $I - A$ (b) $A - I$

(c) $A$ (d) $A + I$

**4.** If the points $(3, -2)$, $(x, 2)$, $(8, 8)$ are collinear, then find the value of $x$.

(a) 2 (b) 3 (c) 4 (d) 5

**5.** If $A=\begin{bmatrix} -2 & 4 \\ -1 & 2 \end{bmatrix}$, then $A^3$ is

(a) null matrix (b) unit matrix

(c) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

**6.** The minor of 6 and cofactor of 4 in the determinant, $\Delta=\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$ are respectively

(a) 6, 6 (b) 6, –6 (c) –6, –6 (d) –6, 6

**7.** If $\begin{bmatrix} x-y & 2x+z \\ 2x-y & 3z+w \end{bmatrix}=\begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$, then the value of $w$ is

(a) 1 (b) 2 (c) 3 (d) 4

**8.** If $\begin{bmatrix} 1 & 2 & 4 \\ 1 & 3 & 5 \\ 1 & 4 & a \end{bmatrix}$ is singular, then the value of $a$ is

(a) $a = -6$ (b) $a = 5$
(c) $a = -5$ (d) $a = 6$

**9.** Suppose that system of equations $x = cy + bz + 1$, $y = az + cx + 2$, $z = bx + ay$ is inconsistent. Then $a^2 + b^2 + c^2 + 2abc =$

(a) 2 (b) $-1$ (c) 0 (d) 1

**10.** If $A = \begin{bmatrix} 2 & 0 & 0 \\ -1 & 2 & 3 \\ 3 & 3 & 5 \end{bmatrix}$, then $A$ (adj $A$) =

(a) $3I$ (b) $2I$ (c) $I$ (d) $4I$

**11.** If $\Delta = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$, then find the value of $|\Delta|$.

(a) 11 (b) $-11$ (c) $-28$ (d) 28

**12.** If $A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$, then find the value of $\alpha$ for which $A^2 = B$.

(a) 1 (b) $-1$
(c) 4 (d) No real values

**13.** If $A$ is a $m \times n$ matrix such that $AB$ and $BA$ are both defined, then $B$ is a

(a) $m \times n$ matrix (b) $n \times m$ matrix
(c) $n \times n$ matrix (d) $m \times m$ matrix

**14.** If $\Delta = \begin{vmatrix} Ax^2 & x^3 & 1 \\ By^2 & y^3 & 1 \\ Cz^2 & z^3 & 1 \end{vmatrix}$ and $\Delta_1 = \begin{vmatrix} Ax & By & Cz \\ x^2 & y^2 & z^2 \\ yz & zx & xy \end{vmatrix}$, then

(a) $\Delta + \Delta_1 = 0$ (b) $\Delta \neq \Delta_1$
(c) $\Delta = x\Delta_1$ (d) $\Delta - \Delta_1 = 0$

**15.** If $A = \begin{bmatrix} 5 & x \\ y & 0 \end{bmatrix}$ and $A = A^T$, then

(a) $x = 0, y = 5$ (b) $x = 2, y = 5$
(c) $x = y$ (d) $x = 0, y = 9$

**16.** If for any $2 \times 2$ square matrix $A$, $A(\text{adj } A) = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$, then the value of $|A|$ is

(a) $-8$ (b) 8 (c) 4 (d) 2

**17.** The system of linear equations $x + y + z = 2$, $2x + y - z = 3$, $3x + 2y + kz = 4$ has a unique solution, if $k$ is not equal to

(a) 4 (b) $-4$ (c) 0 (d) 3

**18.** If $A = \begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & -1 \\ 3 & 1 & 0 \end{bmatrix}$, then $A$ is a

(a) symmetric matrix
(b) skew-symmetric matrix
(c) diagonal matrix
(d) none of these

**Questions number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) options as given below.**

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is **not** the correct explanation of the Assertion (A).
(c) Assertion (A) is true, but Reason (R) is false.
(d) Assertion (A) is false, but Reason (R) is true.

**19. Assertion (A) :** The determinant of a matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & 3 \\ -2 & -3 & 0 \end{bmatrix}$ is zero.

**Reason (R) :** The determinant of every skew-symmetric matrix of odd order is zero.

**20. Assertion (A) :** If $\begin{bmatrix} xy & 4 \\ z+5 & x+y \end{bmatrix}$ and $\begin{bmatrix} 4 & w \\ 0 & 4 \end{bmatrix}$ are equal matrices, then $x = 2$, $y = 2$, $z = -5$ and $w = 4$.

**Reason (R) :** Two matrices are equal, if their orders are same and their corresponding elements are equal.

## SECTION - B

**This section comprises of 5 Very Short Answer (VSA) type questions of 2 marks each.** **$5 \times 2 = 10$**

**21.** Use matrix method to show that the system of equations $2x + 5y = 1$ and $3x + 2y = 7$ is consistent.

**22. (a)** If $x$ is a real root of the equation $\begin{vmatrix} 1 & x & x \\ x & 1 & x \\ x & x & 1 \end{vmatrix} + \begin{vmatrix} 1-x & 1 & 1 \\ 1 & 1-x & 1 \\ 1 & 1 & 1-x \end{vmatrix} = 0$, then find the value of $x^{2007} + x^{-2007}$.

**OR**

**(b)** If the points $(a, 0)$, $(0, b)$ and $(1, 1)$ are collinear, then prove that $a + b = ab$.

**23.** If matrix $\begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$ is a skew-symmetric matrix, then find the values of $a$, $b$ and $c$.

**24.** **(a)** Given $A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$, compute $A^{-1}$ and show that $2A^{-1} = 9I - A$.

**OR**

**(b)** If the matrix $A = \begin{bmatrix} 1 & -2 & 3 \\ 1 & 2 & 1 \\ x & 2 & -3 \end{bmatrix}$ is singular, then find $x$.

**25.** For what values of $x$ and $y$, the following matrices are equal?

$$A = \begin{bmatrix} 2x+1 & 3y \\ 0 & y^2 - 5y \end{bmatrix}, B = \begin{bmatrix} x+3 & y^2+2 \\ 0 & -6 \end{bmatrix}$$

## SECTION - C

**This section comprises of 6 Short Answer (SA) Type questions of 3 marks each.** **6 × 3 = 18**

**26.** **(a)** Solve the system of linear equations using matrix method $5x + 2y = 4$ and $7x + 3y = 5$.

**OR**

**(b)** If $A = \begin{bmatrix} -1 & 2 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, then show that $A^{-1} = A^2$.

**27.** Find $A$, if $\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} A = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}$.

**28.** Write minor and cofactor of the element $a_{11}$, $a_{13}$ and $a_{31}$ of the determinant $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$.

**29.** **(a)** Given $A = \begin{bmatrix} 2 & 4 & 0 \\ 3 & 9 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 2 & 8 \\ 1 & 3 \end{bmatrix}$

Is $(AB)' = B'A'$?

**OR**

**(b)** Find the inverse of matrix $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$.

**30.** If the coordinates of the vertices of an equilateral triangle with sides of length '$a$' are $(x_1, y_1)$, $(x_2, y_2)$ and $(x_3, y_3)$, then show that $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2 = \frac{3a^4}{4}$.

**31.** **(a)** Find the values of $x$, $y$ and $z$ if $\begin{bmatrix} -2x+y \\ x+y+z \\ x+y \end{bmatrix} = \begin{bmatrix} -3 \\ 3 \\ 3 \end{bmatrix}$.

**OR**

**(b)** If $\begin{bmatrix} x^2 - 4x & x^2 \\ x^2 & x^3 \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ -x+2 & 1 \end{bmatrix}$, then find $x$.

## SECTION - D

**This section comprises of 4 Long Answer (LA) Type questions of 5 marks each.** **4 × 5 = 20**

**32.** **(a)** Find $A^{-1}$, if $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ and show that $A^{-1} = \frac{A^2 - 3I}{2}$.

**OR**

**(b)** Find $x, y$ and $z$, if $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfies $A' = A^{-1}$.

**33.** If $A = \begin{bmatrix} 3 & 1 \\ 7 & 2 \end{bmatrix}$, then find $A^{-1}$ and use this to solve the simultaneous equations $3x + 7y = 4$ and $x + 2y = 1$.

**34.** Express the matrix $A = \begin{bmatrix} 2 & 4 & -6 \\ 7 & 3 & 5 \\ 1 & -2 & 4 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix.

**35.** **(a)** Find the equation of line joining $P(11, 7)$ and $Q(5, 5)$ using determinant. Also, find the value of $k$, if $R(-1, k)$ is the point such that area of $\Delta PQR$ is 9 sq. units.

**OR**

**(b)** Using matrix method, solve the following system of equations : $x + y - z = 3$; $2x + 3y + z = 10$; $3x - y - 7z = 1$.

## SECTION - E

**This section comprises of 3 case study / passage based questions of 4 marks each.** **$3 \times 4 = 12$**

**36.** To promote the making of toilets for women, an organisation tried to generate awareness through (1) house calls (2) emails and (3) announcements. The cost for each mode per attempt is given below :

(1) ₹ 50 (2) ₹ 20 (3) ₹ 40

The number of attempts made in the villages $X$, $Y$ and $Z$ are given below :

| | (1) | (2) | (3) |
|---|---|---|---|
| $X$ | 400 | 300 | 100 |
| $Y$ | 300 | 250 | 75 |
| $Z$ | 500 | 400 | 150 |

Also, the chance of making of toilets corresponding to one attempt of given modes is

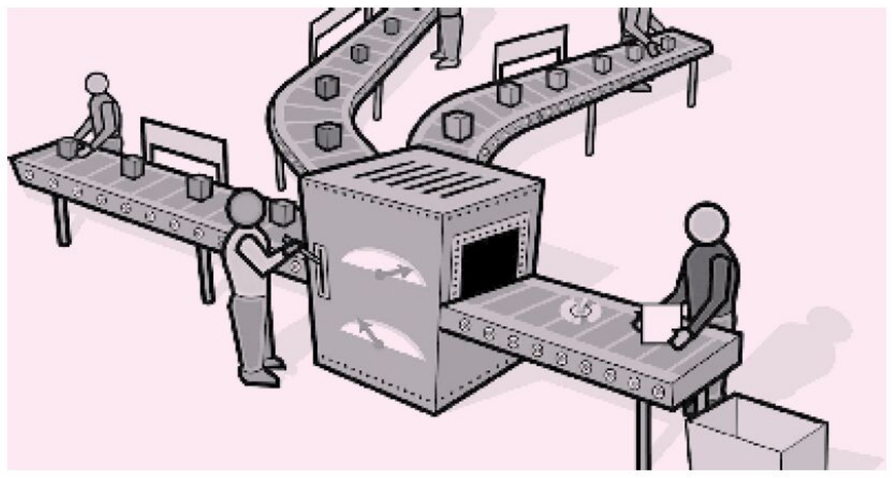
(1) 2% (2) 4% (3) 20%

Based on the above information, answer the following questions :

(i) Find the cost incurred by the organisation on village $X$.

(ii) Find the total number of toilets that can be expected after the promotion in village $X$.

**37.** A company produces three products every day. Their production on certain day is 45 tons. It is found that the production of third product exceeds the production of first product by 8 tons while the total production of first and third product is twice the production of second product.



Based on the above information, answer the following questions :

(i) Convert the above situation into a matrix equaiton of the form $AX = B$.

(ii) Check whether the system $AX = B$ is consistent or inconsistent.

(iii) (a) Find the inverse of $A$ using $A^{-1} = \dfrac{\text{adj}\, A}{|A|}$.

**OR**

(iii) (b) Find $A^{-1}$ if $A$ satisfies $AA' = I$.

**38.** A company encrypts secret messages using matrices. They use an encryption matrix $A$ and a message matrix $M$ to encode the message.
The company uses the matrices:

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \text{ and } M = \begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix}$$

Based on the above information, answer the following questions :

(i) Find the value of $AM$.

(ii) Find the value of $MA$.

(iii) (a) If there exists an other matrix $C$ such that $C - (A + M) = A$, then find the value of the matrix $C$.

**OR**

(iii) (b) If $D = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ such that $D = AM - A$, then find the value of $a + b + c + d$.

## SOLUTIONS

**1.** **(c) :** Since, $A$ is a symmetric matrix $\Rightarrow A^T = A$

$$\Rightarrow \begin{bmatrix} 3 & 2x+3 \\ x-1 & x+2 \end{bmatrix} = \begin{bmatrix} 3 & x-1 \\ 2x+3 & x+2 \end{bmatrix}$$

$\Rightarrow x - 1 = 2x + 3 \Rightarrow x = -4$

**2.** **(d) :** We have, $|A| = x - 2x(x - 1) = -9$

$\Rightarrow x - 2x^2 + 2x = -9 \Rightarrow 2x^2 - 3x - 9 = 0$

$$\Rightarrow x = \frac{3 \pm \sqrt{9+72}}{4} = \frac{3 \pm 9}{4} \Rightarrow x = 3 \text{ or } \frac{-3}{2}$$

**3.** **(a) :** If $A$ is any square matrix, then $AA^{-1} = I$ and $A^{-1}I = A^{-1}$

Since, $A^2 - A + I = O \Rightarrow A^{-1}A^2 - A^{-1}A + A^{-1}I = O$

$\Rightarrow (A^{-1}A)A - (A^{-1}A) + A^{-1} = O \Rightarrow IA - I + A^{-1} = O$

$\Rightarrow A - I + A^{-1} = O \Rightarrow A^{-1} = I - A$

**4.** **(d) :** Since, the given points are collinear.

$$\therefore \frac{1}{2}\begin{vmatrix} 3 & -2 & 1 \\ x & 2 & 1 \\ 8 & 8 & 1 \end{vmatrix} = 0 \Rightarrow 3(2-8) + 2(x-8) + 1(8x-16) = 0$$

$\Rightarrow -18 + 2x - 16 + 8x - 16 = 0 \Rightarrow 10x = 50 \Rightarrow x = 5$

**5.** **(a) :** Given that, $A=\begin{bmatrix}-2 & 4\\ -1 & 2\end{bmatrix}$

$\therefore \quad A^2=\begin{bmatrix}-2 & 4\\ -1 & 2\end{bmatrix}\begin{bmatrix}-2 & 4\\ -1 & 2\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$

$\Rightarrow$ $A^3$ is a null matrix.

**6.** **(d) :** Minor of $6 = M_{23} = \begin{vmatrix}1 & 2\\ 7 & 8\end{vmatrix} = 8 - 14 = -6$

Cofactor of $4 = c_{21} = (-1)^{2+1}\begin{vmatrix}2 & 3\\ 8 & 9\end{vmatrix} = -(18-24) = 6$

**7.** **(d) :** Since, $\begin{bmatrix}x-y & 2x+z\\ 2x-y & 3z+w\end{bmatrix}=\begin{bmatrix}-1 & 5\\ 0 & 13\end{bmatrix}$

$\Rightarrow \quad x - y = -1$ ...(i), $\quad 2x - y = 0$ ...(ii)

$2x + z = 5$ ...(iii) and $3z + w = 13$ ...(iv)

On solving these equations, we get

$x = 1, y = 2, z = 3$ and $w = 4$

**8.** **(d) :** Let $A = \begin{bmatrix}1 & 2 & 4\\ 1 & 3 & 5\\ 1 & 4 & a\end{bmatrix}$

Since, $A$ is singular matrix,

$\Rightarrow \quad |A| = 0 \Rightarrow \begin{vmatrix}1 & 2 & 4\\ 1 & 3 & 5\\ 1 & 4 & a\end{vmatrix} = 0$

$\Rightarrow \quad 1(3a - 20) - 2(a - 5) + 4(4 - 3) = 0 \Rightarrow a = 6$

**9.** **(d) :** The system is inconsistent if $\begin{vmatrix}-1 & c & b\\ c & -1 & a\\ b & a & -1\end{vmatrix} = 0$

$\Rightarrow \quad -1(1 - a^2) - c(-c - ab) + b(ca + b) = 0$

$\Rightarrow \quad a^2 + b^2 + c^2 + 2abc = 1$

**10.** **(b) :** Given, $A = \begin{bmatrix}2 & 0 & 0\\ -1 & 2 & 3\\ 3 & 3 & 5\end{bmatrix}$

$\therefore \quad A(\text{adj } A) = |A|\, I = \begin{vmatrix}2 & 0 & 0\\ -1 & 2 & 3\\ 3 & 3 & 5\end{vmatrix} I$

$= (2(10 - 9) - 0 + 0)\, I = 2(1)I = 2I$

**11.** **(d) :** Here, $\Delta = \begin{vmatrix}2 & -3 & 5\\ 6 & 0 & 4\\ 1 & 5 & -7\end{vmatrix}$

$\Rightarrow \quad \Delta = 2(-20) - 3(46) + 5(30) = -28 \Rightarrow |\Delta| = 28$

**12.** **(d) :** Given that, $A=\begin{bmatrix}\alpha & 0\\ 1 & 1\end{bmatrix}$

$\Rightarrow \quad A^2=\begin{bmatrix}\alpha & 0\\ 1 & 1\end{bmatrix}\begin{bmatrix}\alpha & 0\\ 1 & 1\end{bmatrix}$

$=\begin{bmatrix}\alpha^2+0 & 0+0\\ \alpha+1 & 0+1\end{bmatrix}=\begin{bmatrix}\alpha^2 & 0\\ \alpha+1 & 1\end{bmatrix}$

Also, $B = A^2 \Rightarrow \begin{bmatrix}1 & 0\\ 5 & 1\end{bmatrix}=\begin{bmatrix}\alpha^2 & 0\\ \alpha+1 & 1\end{bmatrix}$

Clearly, this is not satisfied by any real value of $\alpha$.

**13.** **(b) :** Since $AB$ exists, therefore, number of columns in $A$ = number of rows in $B$. So, $B$ has $n$ rows. Since $BA$ exists, therefore, number of columns in $B$ = number of rows in $A$. So, $B$ has $m$ columns.

$\therefore \quad B$ is a $n \times m$ matrix.

**14.** **(d) :** Expand $\Delta$ along first column, we get

$\Delta = Ax^2(y^3 - z^3) - By^2(x^3 - z^3) + Cz^2(x^3 - y^3)$ ...(i)

Now, expand $\Delta_1$ along first row, we get

$\Delta_1 = Ax(xy^3 - xz^3) - By(x^3y - yz^3) + Cz(x^3z - zy^3)$

$= Ax^2(y^3 - z^3) - By^2(x^3 - z^3) + Cz^2(x^3 - y^3)$ ...(ii)

From (i) & (ii), we get

$\Delta = \Delta_1 \Rightarrow \Delta - \Delta_1 = 0$

**15.** **(c) :** We have, $A=\begin{bmatrix}5 & x\\ y & 0\end{bmatrix} \Rightarrow A^T=\begin{bmatrix}5 & y\\ x & 0\end{bmatrix}$

Now, $A = A^T \Rightarrow \begin{bmatrix}5 & x\\ y & 0\end{bmatrix}=\begin{bmatrix}5 & y\\ x & 0\end{bmatrix} \Rightarrow x = y$

**16.** **(b) :** We have, $A(\text{adj } A)=\begin{bmatrix}8 & 0\\ 0 & 8\end{bmatrix}$

$\Rightarrow \quad |A|\, I = 8\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ $\quad [\because \ A(\text{adj } A) = |A|I]$

$\therefore \quad |A| = 8$

**17.** **(c) :** The given system has a unique solution, if

$\begin{vmatrix}1 & 1 & 1\\ 2 & 1 & -1\\ 3 & 2 & k\end{vmatrix} \neq 0$

$\Rightarrow \quad 1(k+2) - 1(2k+3) + 1\,(4-3) \neq 0$

$\Rightarrow \quad k + 2 - 2k - 3 + 1 \neq 0 \Rightarrow k \neq 0$

**18. (b) :** We have,

$$A^T = \begin{bmatrix} 0 & -2 & 3 \\ 2 & 0 & 1 \\ -3 & -1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & -1 \\ 3 & 1 & 0 \end{bmatrix} = -A$$

$\therefore$ $A$ is a skew-symmetric matrix.

**19. (a) :** Given, $A = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & 3 \\ -2 & -3 & 0 \end{bmatrix}$

$$A' = \begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & -3 \\ 2 & 3 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & 3 \\ -2 & -3 & 0 \end{bmatrix} = -A$$

$\therefore$ Matrix is skew symmetric matrix of an odd order.

$\therefore \quad |A| = 0$

Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

**20. (a) :** We have, $\begin{bmatrix} xy & 4 \\ z+5 & x+y \end{bmatrix} = \begin{bmatrix} 4 & w \\ 0 & 4 \end{bmatrix}$

On equating the corresponding elements, we get

$xy = 4$, $w = 4$, $z + 5 = 0$ and $x + y = 4$

On solving these equations, we get $x = 2$, $y = 2$, $z = -5$ and $w = 4$. Also, the two matrices are equal, if their orders are same and their corresponding elements are equal.

$\therefore$ Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

**21.** The given system of equations can be written as

$$\begin{bmatrix} 2 & 5 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \end{bmatrix} \textit{i.e.}, AX = B, \text{ where } A = \begin{bmatrix} 2 & 5 \\ 3 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}$$

and $B = \begin{bmatrix} 1 \\ 7 \end{bmatrix}$

Now, $|A| = \begin{vmatrix} 2 & 5 \\ 3 & 2 \end{vmatrix} = 4 - 15 = -11 \neq 0$

$\therefore A^{-1}$ exists and hence system of equations is consistent.

**22. (a)** Expanding the two determinant along $R_1$, we get

$(1 - 3x^2 + 2x^3) + (3x^2 - x^3) = 0$

$\Rightarrow \quad x^3 + 1 = 0 \Rightarrow (x + 1)(x^2 - x + 1) = 0$

$\Rightarrow x = -1, \dfrac{1 \pm \sqrt{3}i}{2}$ (Neglect)

$\therefore \quad x^{2007} + x^{-2007} = -1 - 1 = -2$

**OR**

**(b)** Since, the points $(a, 0)$, $(0, b)$ and $(1, 1)$ are collinear.

$$\therefore \quad \begin{vmatrix} a & 0 & 1 \\ 0 & b & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

$\Rightarrow \quad a(b - 1) - 0(0 - 1) + 1(0 - b) = 0$ [Expanding along $R_1$]

$\Rightarrow \quad ab - a - b = 0 \Rightarrow ab = a + b$

Hence proved.

**23.** Given, $A = \begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$ is a skew-symmetric matrix

$\therefore \quad A' = -A$

$$\Rightarrow \begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}' = -\begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 2 & c \\ a & b & 1 \\ 3 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -a & -3 \\ -2 & -b & 1 \\ -c & -1 & 0 \end{bmatrix}$$

On equating the corresponding elements, we get

$a = -2$ and $b = -b \Rightarrow 2b = 0 \Rightarrow b = 0$ and $c = -3$

**24. (a)** We have, $A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix} \Rightarrow |A| = 14 - 12 = 2 \neq 0$

$\therefore \quad A^{-1}$ exists.

So, $A^{-1} = \dfrac{1}{|A|}\,\text{adj}\,A = \dfrac{1}{2}\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix} \Rightarrow 2A^{-1} = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$ ...(i)

Now, $9I - A = 9\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix} = \begin{bmatrix} 9-2 & 0+3 \\ 0+4 & 9-7 \end{bmatrix}$

$= \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix} = 2A^{-1}$ [Using (i)]

**OR**

**(b)** We have, $A = \begin{bmatrix} 1 & -2 & 3 \\ 1 & 2 & 1 \\ x & 2 & -3 \end{bmatrix}$

For singular, $|A| = 0 \quad \Rightarrow \begin{vmatrix} 1 & -2 & 3 \\ 1 & 2 & 1 \\ x & 2 & -3 \end{vmatrix} = 0$

Expanding along first row, we get

$1(-6 - 2) + 2(-3 - x) + 3(2 - 2x) = 0$

$\Rightarrow \quad -8 - 6 - 2x + 6 - 6x = 0 \Rightarrow 8x = -8 \Rightarrow x = -1$

**25.** By definition of equal matrices, we have

$2x + 1 = x + 3$

$\Rightarrow \quad x = 2;\ 3y = y^2 + 2$

$\Rightarrow \quad y^2 - 3y + 2 = 0 \Rightarrow (y - 1)(y - 2) = 0 \Rightarrow y = 1, 2;$

$y^2 - 5y = -6 \Rightarrow y^2 - 5y + 6 = 0$

$\Rightarrow \quad (y - 2)(y - 3) = 0 \Rightarrow y = 2, 3$

$\therefore \quad x = 2, y = 1, 2$ and $y = 2, 3$

$\Rightarrow \quad x = 2, y = 2$

**26.** **(a) :** The given system of equations can be written as $\begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$ *i.e.*, $AX = B$,

where $A = \begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

Now, $|A| = \begin{vmatrix} 5 & 2 \\ 7 & 3 \end{vmatrix} = 15 - 14 = 1 \neq 0$

$\therefore \quad A^{-1}$ exists.

$\therefore$ The system of equation has unique solution which is given by $X = A^{-1}B$.

$A_{11} = 3, A_{12} = -7, A_{21} = -2, A_{22} = 5$

$$\therefore \quad \text{adj } A = \begin{bmatrix} 3 & -7 \\ -2 & 5 \end{bmatrix}' = \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\ \text{adj } A = \frac{1}{1}\begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}$$

$\therefore \quad$ Solution is given by, $X = A^{-1}B = \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}\begin{bmatrix} 4 \\ 5 \end{bmatrix}$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 - 10 \\ -28 + 25 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

$\therefore \quad x = 2, y = -3$

**OR**

**(b)** By definition, a matrix $B$ is an inverse of $A$ if $AB = I = BA$

Here, $A^2$ is inverse of $A$.

$\therefore \quad$ It is sufficient to prove that

$A^2A = I = A \cdot A^2$ *i.e.*, $A^3 = I$.

$$\text{Now, } A^2 = A \cdot A = \begin{bmatrix} -1 & 2 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 2 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1-2+0 & -2+2+0 & 0+2+0 \\ 1-1+0 & -2+1+1 & 0+1+0 \\ 0-1+0 & 0+1+0 & 0+1+0 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 2 \\ 0 & 0 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$

$$\therefore \quad A^3 = A^2 \cdot A = \begin{bmatrix} -1 & 0 & 2 \\ 0 & 0 & 1 \\ -1 & 1 & 1 \end{bmatrix}\begin{bmatrix} -1 & 2 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0+0 & -2+0+2 & 0+0+0 \\ 0+0+0 & 0+0+1 & 0+0+0 \\ 1-1+0 & -2+1+1 & 0+1+0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

Hence, $A^{-1} = A^2$.

**27.** Let $A = [x \quad y \quad z]$ as order of $A$ will be $1 \times 3$.

$$\text{Given, } \begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix}_{3\times1} \quad A = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}_{3\times3}$$

$$\therefore \quad \begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix}_{3\times1} \begin{bmatrix} x & y & z \end{bmatrix}_{1\times3} = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}_{3\times3}$$

$$\Rightarrow \quad \begin{bmatrix} 4x & 4y & 4z \\ x & y & z \\ 3x & 3y & 3z \end{bmatrix} = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}$$

On comparing the corresponding elements of two matrices, we get

$4x = -4 \Rightarrow x = -1$

$4y = 8 \Rightarrow y = 2$

$4z = 4 \Rightarrow z = 1$

Hence, $A = [-1 \quad 2 \quad 1]$

**28.** We have, $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

$$\text{Minor of } a_{11} = M_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} = a_{22}a_{33} - a_{23}a_{32}$$

$$\text{Minor of } a_{13} = M_{13} = \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} = a_{21}a_{32} - a_{22}a_{31}$$

$$\text{Minor of } a_{31} = M_{31} = \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} = a_{12}a_{23} - a_{13}a_{22}$$

$$\text{Also, cofactor of } a_{11} = A_{11} = (-1)^{1+1}(a_{22}a_{33} - a_{23}a_{32})$$

$$= a_{22}a_{33} - a_{23}a_{32}$$

Cofactor of $a_{13} = A_{13} = (-1)^{1+3}(a_{21}a_{32} - a_{22}a_{31})$
$= a_{21}a_{32} - a_{22}a_{31}$

Cofactor of $a_{31} = A_{31} = (-1)^{3+1}(a_{12}a_{23} - a_{13}a_{22})$
$= a_{12}a_{23} - a_{13}a_{22}$

**29. (a)** We have, $A = \begin{bmatrix} 2 & 4 & 0 \\ 3 & 9 & 6 \end{bmatrix}_{2\times3}$ and $B = \begin{bmatrix} 1 & 4 \\ 2 & 8 \\ 1 & 3 \end{bmatrix}_{3\times2}$

Now, $AB = \begin{bmatrix} 2+8+0 & 8+32+0 \\ 3+18+6 & 12+72+18 \end{bmatrix} = \begin{bmatrix} 10 & 40 \\ 27 & 102 \end{bmatrix}$

$\Rightarrow (AB)' = \begin{bmatrix} 10 & 27 \\ 40 & 102 \end{bmatrix}$ ...(i)

Also, $B' = \begin{bmatrix} 1 & 2 & 1 \\ 4 & 8 & 3 \end{bmatrix}_{2\times3}$ and $A' = \begin{bmatrix} 2 & 3 \\ 4 & 9 \\ 0 & 6 \end{bmatrix}_{3\times2}$

$\therefore B'A' = \begin{bmatrix} 2+8+0 & 3+18+6 \\ 8+32+0 & 12+72+18 \end{bmatrix} = \begin{bmatrix} 10 & 27 \\ 40 & 102 \end{bmatrix}$ ...(ii)

From (i) and (ii), we get $(AB)' = B'A'$.

**OR**

**(b)** Let $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$\therefore \quad |A| = \begin{vmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{vmatrix} = \cos^2\theta + \sin^2\theta = 1, \forall\ \theta$

Since, $|A| \neq 0 \quad \therefore A^{-1}$ exists

Now, $A_{11} = \cos\theta$; $A_{12} = -(-\sin\theta) = \sin\theta$,

$A_{21} = -\sin\theta$; $A_{22} = \cos\theta$

$$\text{adj } A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}' = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\text{adj } A = \frac{1}{1}\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

**30.** As we know that area of a triangle with vertices $(x_1, y_1)$, $(x_2, y_2)$ and $(x_3, y_3)$ is given by

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \Rightarrow \Delta^2 = \frac{1}{4}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2 \quad \text{...(i)}$$

Since, area of an equilateral triangle with side $a$ is $\Delta = \frac{\sqrt{3}}{4}a^2$.

$$\Rightarrow \quad \Delta^2 = \frac{3}{16}a^4 \quad \text{...(ii)}$$

From (i) and (ii), we get

$$\frac{3}{16}a^4 = \frac{1}{4}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2 \Rightarrow \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2 = \frac{3}{4}a^4$$

**31. (a)** We have, $\begin{bmatrix} -2x+y \\ x+y+z \\ x+y \end{bmatrix} = \begin{bmatrix} -3 \\ 3 \\ 3 \end{bmatrix}$

Equating the corresponding elements of two matrices, we get

$-2x + y = -3$ ...(i) ; $x + y + z = 3$ ...(ii) ; $x + y = 3$ ...(iii)

From (ii) and (iii), we get

$3 + z = 3 \Rightarrow z = 0$

Solving (i) and (iii), we get

$-3x = -6 \Rightarrow x = 2$

From (iii), $2 + y = 3 \Rightarrow y = 1$

$\therefore \quad x = 2, y = 1, z = 0$

**OR**

**(b)** We have, $\begin{bmatrix} x^2 - 4x & x^2 \\ x^2 & x^3 \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ -x+2 & 1 \end{bmatrix}$

Equating the corresponding elements of two matrices, we get

$x^2 - 4x = -3 \Rightarrow x^2 - 4x + 3 = 0$

$\Rightarrow \quad (x - 1)(x - 3) = 0$

$\Rightarrow \quad x = 1, 3$

$x^2 = 1 \Rightarrow x = \pm 1$

$x^2 = -x + 2$

$\Rightarrow x^2 + x - 2 = 0$

$\Rightarrow \quad (x - 1)(x + 2) = 0$

$\Rightarrow \quad x = 1, -2$

$x^3 = 1 \Rightarrow x^3 - 1 = 0$

$\Rightarrow (x - 1)(x^2 + x + 1) = 0$

$$\Rightarrow \quad x = 1, \frac{-1 \pm \sqrt{3}\,i}{2}$$

Since, common value of $x$ is 1.

$\therefore \quad x = 1$

**32. (a)** Given, $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

$\therefore \quad |A| = (-1)(0 - 1) + 1(1 - 0) = 1 + 1 = 2 \neq 0$

$\therefore \quad A^{-1}$ exists

$A_{11} = -1$, $A_{12} = 1$, $A_{13} = 1$, $A_{21} = 1$, $A_{22} = -1$, $A_{23} = 1$, $A_{31} = 1$, $A_{32} = 1$ and $A_{33} = -1$

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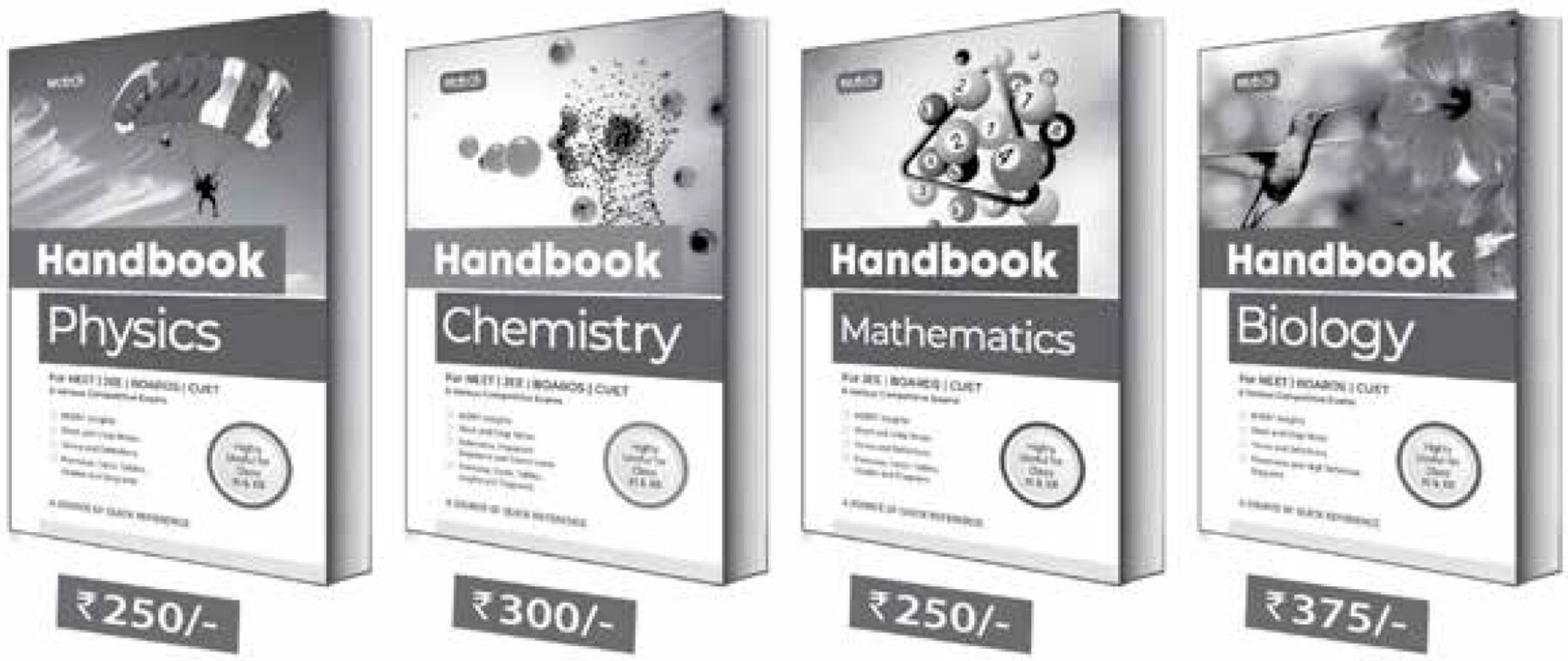


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$$\therefore \quad \text{adj } A = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}' = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

$$\text{Hence, } A^{-1} = \frac{1}{|A|}\text{adj } A = \frac{1}{2}\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} \qquad \ldots(i)$$

$$\text{Also, } A^2 = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\text{Now, } \frac{A^2 - 3I}{2} = \frac{1}{2}\left[\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right]$$

$$= \frac{1}{2}\begin{bmatrix} 2-3 & 1-0 & 1-0 \\ 1-0 & 2-3 & 1-0 \\ 1-0 & 1-0 & 2-3 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} = A^{-1}$$

[Using (i)]

Hence proved.

**OR**

**(b)** We have,

$$A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} \text{ then } A' = \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}$$

Since, $A' = A^{-1}$

$\therefore \quad AA' = AA^{-1}$ [Multiplying by $A$ on both sides]

$\Rightarrow \quad AA' = I$

$$\Rightarrow \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}\begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4y^2+z^2 & 2y^2-z^2 & -2y^2+z^2 \\ 2y^2-z^2 & x^2+y^2+z^2 & x^2-y^2-z^2 \\ -2y^2+z^2 & x^2-y^2-z^2 & x^2+y^2+z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow \quad 2y^2 - z^2 = 0$

$\Rightarrow \quad 2y^2 = z^2 \qquad \ldots(i)$

and $4y^2 + z^2 = 1$

$\Rightarrow \quad 2 \cdot z^2 + z^2 = 1$ [Using (i)]

$\Rightarrow \quad z = \pm\frac{1}{\sqrt{3}} \quad \because \quad y^2 = \frac{z^2}{2} \Rightarrow y = \pm\frac{1}{\sqrt{6}}$

Also, $x^2 + y^2 + z^2 = 1$

$\Rightarrow \quad x^2 = 1 - y^2 - z^2 = 1 - \frac{1}{6} - \frac{1}{3} = \frac{1}{2}$

$\Rightarrow \quad x = \pm\frac{1}{\sqrt{2}}$

$\therefore \; x = \pm\frac{1}{\sqrt{2}}, y = \pm\frac{1}{\sqrt{6}}$ and $z = \pm\frac{1}{\sqrt{3}}$

**33.** Given, $A = \begin{bmatrix} 3 & 1 \\ 7 & 2 \end{bmatrix}$

Then, $|A| = 6 - 7 = -1 \neq 0 \quad \therefore \quad A^{-1}$ exists

$A_{11} = 2, A_{12} = -7, A_{21} = -1, A_{22} = 3$

$$\therefore \quad \text{adj } A = \begin{bmatrix} 2 & -7 \\ -1 & 3 \end{bmatrix}' = \begin{bmatrix} 2 & -1 \\ -7 & 3 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{|A|}\text{adj } A = \frac{1}{-1}\begin{bmatrix} 2 & -1 \\ -7 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 7 & -3 \end{bmatrix}$$

The given system of equation is

$3x + 7y = 4$

$x + 2y = 1$

The system can be written as

$$\begin{bmatrix} 3 & 7 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix} \text{ i.e., } A'X = B, \text{ where}$$

$$A' = \begin{bmatrix} 3 & 7 \\ 1 & 2 \end{bmatrix}, \; X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

Now, $|A'| = -1 \neq 0$

$\therefore$ The given system has a unique solution given by

$X = (A')^{-1}B$

$\Rightarrow \quad X = (A^{-1})'B$ $\qquad [\because (A')^{-1} = (A^{-1})']$

$$\Rightarrow X = \begin{bmatrix} -2 & 1 \\ 7 & -3 \end{bmatrix}'\begin{bmatrix} 4 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 & 7 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \end{bmatrix} = \begin{bmatrix} -8+7 \\ 4-3 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$\Rightarrow \quad x = -1, y = 1$

Hence, the solution of given system of equations is $x = -1, y = 1$.

**34.** We have, $A = \begin{bmatrix} 2 & 4 & -6 \\ 7 & 3 & 5 \\ 1 & -2 & 4 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 2 & 7 & 1 \\ 4 & 3 & -2 \\ -6 & 5 & 4 \end{bmatrix}$

Let $P=\frac{1}{2}(A+A')=\frac{1}{2}\left\{\begin{bmatrix}2&4&-6\\7&3&5\\1&-2&4\end{bmatrix}+\begin{bmatrix}2&7&1\\4&3&-2\\-6&5&4\end{bmatrix}\right\}$

$=\frac{1}{2}\begin{bmatrix}4&11&-5\\11&6&3\\-5&3&8\end{bmatrix}=\begin{bmatrix}2&\frac{11}{2}&\frac{-5}{2}\\\frac{11}{2}&3&\frac{3}{2}\\\frac{-5}{2}&\frac{3}{2}&4\end{bmatrix}$,

which is a symmetric matrix.

Also, $Q=\frac{1}{2}(A-A')=\frac{1}{2}\left\{\begin{bmatrix}2&4&-6\\7&3&5\\1&-2&4\end{bmatrix}-\begin{bmatrix}2&7&1\\4&3&-2\\-6&5&4\end{bmatrix}\right\}$

$=\frac{1}{2}\begin{bmatrix}0&-3&-7\\3&0&7\\7&-7&0\end{bmatrix}=\begin{bmatrix}0&\frac{-3}{2}&\frac{-7}{2}\\\frac{3}{2}&0&\frac{7}{2}\\\frac{7}{2}&\frac{-7}{2}&0\end{bmatrix}$,

which is a skew-symmetric matrix.

Now, $A=\frac{1}{2}(A+A')+\frac{1}{2}(A-A')=P+Q$

$=\begin{bmatrix}2&\frac{11}{2}&\frac{-5}{2}\\\frac{11}{2}&3&\frac{3}{2}\\\frac{-5}{2}&\frac{3}{2}&4\end{bmatrix}+\begin{bmatrix}0&\frac{-3}{2}&\frac{-7}{2}\\\frac{3}{2}&0&\frac{7}{2}\\\frac{7}{2}&\frac{-7}{2}&0\end{bmatrix}$

Hence, in above equation, $A$ is represented as sum of symmetric and skew-symmetric matrix.

**35. (a)** Let $A(x, y)$ be any point on line $PQ$. Then, the points $P$, $Q$ and $A$ are collinear.

So, $\begin{vmatrix}11&7&1\\5&5&1\\x&y&1\end{vmatrix}=0$

$\Rightarrow \quad 11(5-y)-7(5-x)+1(5y-5x)=0$

$\Rightarrow \quad 55-11y-35+7x+5y-5x=0$

$\Rightarrow x-3y+10=0,$

which is the required equation of line joining points $P$ and $Q$.

Now, according to the question, area of $\Delta PQR = 9$ sq. units.

$\therefore \quad \frac{1}{2}\begin{vmatrix}11&7&1\\5&5&1\\-1&k&1\end{vmatrix}=\pm 9$

$\Rightarrow \quad 11(5-k)-7(6)+1(5k+5)=\pm 18$

$\Rightarrow \quad -6k+18=\pm 18$

$\Rightarrow k=\frac{-18\pm 18}{-6}$

For positive sign, $k=\frac{-18+18}{-6}=0$

For negative sign, $k=\frac{-18-18}{-6}=6$

$\therefore$ Required values of $k$ are 0 and 6.

**OR**

**(b)** The given system of equations can be written as

$\begin{bmatrix}1&1&-1\\2&3&1\\3&-1&-7\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}3\\10\\1\end{bmatrix}$ *i.e.*, $AX=B,$

where $A=\begin{bmatrix}1&1&-1\\2&3&1\\3&-1&-7\end{bmatrix}$, $X=\begin{bmatrix}x\\y\\z\end{bmatrix}$ and $B=\begin{bmatrix}3\\10\\1\end{bmatrix}$

Now, $|A|=\begin{vmatrix}1&1&-1\\2&3&1\\3&-1&-7\end{vmatrix}$

$=1(-21+1)-1(-14-3)-1(-2-9)=-20+17+11=8\neq 0$

$\therefore \quad A^{-1}$ exists.

So, the system of equations has a unique solution which given by $X=A^{-1}B$.

$A_{11}=-20, A_{12}=17, A_{13}=-11, A_{21}=8,$

$A_{22}=-4, A_{23}=4, A_{31}=4, A_{32}=-3, A_{33}=1$

Now, $\text{adj } A=\begin{bmatrix}-20&17&-11\\8&-4&4\\4&-3&1\end{bmatrix}'=\begin{bmatrix}-20&8&4\\17&-4&-3\\-11&4&1\end{bmatrix}$

$\therefore \quad A^{-1}=\frac{1}{|A|}\text{adj } A=\frac{1}{8}\begin{bmatrix}-20&8&4\\17&-4&-3\\-11&4&1\end{bmatrix}$

Now, $X=A^{-1}B=\frac{1}{8}\begin{bmatrix}-20&8&4\\17&-4&-3\\-11&4&1\end{bmatrix}\begin{bmatrix}3\\10\\1\end{bmatrix}$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 24 \\ 8 \\ 8 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$$

$\Rightarrow \; x = 3, y = 1, z = 1$

**36.** (i) : Let ₹ $A$, ₹ $B$ and ₹ $C$ be the cost incurred by the organisation for villages $X$, $Y$ and $Z$ respectively. Then $A$, $B$, $C$ will be given by the following matrix equation.

$$\begin{bmatrix} 400 & 300 & 100 \\ 300 & 250 & 75 \\ 500 & 400 & 150 \end{bmatrix}\begin{bmatrix} 50 \\ 20 \\ 40 \end{bmatrix} = \begin{bmatrix} A \\ B \\ C \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 400\times 50 + 300\times 20 + 100\times 40 \\ 300\times 50 + 250\times 20 + 75\times 40 \\ 500\times 50 + 400\times 20 + 150\times 40 \end{bmatrix}$$

$$= \begin{bmatrix} 20000 + 6000 + 4000 \\ 15000 + 5000 + 3000 \\ 25000 + 8000 + 6000 \end{bmatrix} = \begin{bmatrix} 30000 \\ 23000 \\ 39000 \end{bmatrix}$$

(ii) Total number of toilets that can be expected in each village is given by the following matrix

$$\begin{matrix} X \\ Y \\ Z \end{matrix}\begin{bmatrix} 400 & 300 & 100 \\ 300 & 250 & 75 \\ 500 & 400 & 150 \end{bmatrix}\begin{bmatrix} 2/100 \\ 4/100 \\ 20/100 \end{bmatrix} = \begin{matrix} X \\ Y \\ Z \end{matrix}\begin{bmatrix} 8+12+20 \\ 6+10+15 \\ 10+16+30 \end{bmatrix}$$

$$= \begin{matrix} X \\ Y \\ Z \end{matrix}\begin{bmatrix} 40 \\ 31 \\ 56 \end{bmatrix}$$

**37.** (i) According to given condition, we have the following system of linear equations.

$x + y + z = 45$

$x + 8 = z$ or $x + 0\cdot y - z = -8$

and $x + z = 2y$ or $x - 2y + z = 0$

The above system of equations can be written in matrix form as $AX = B$, where

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & -2 & 1 \end{pmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 45 \\ -8 \\ 0 \end{bmatrix}$$

(ii) Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & -2 & 1 \end{bmatrix}$

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & -2 & 1 \end{vmatrix} = 1(0-2) - 1(1+1) + 1(-2-0) = -6 \neq 0$$

$\therefore$ System is consistent.

(iii) (a) : We have, $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & -2 & 1 \end{bmatrix}$ and $|A| = -6$

$$\text{Adj } A = \begin{bmatrix} -2 & -2 & -2 \\ -3 & 0 & 3 \\ -1 & 2 & -1 \end{bmatrix}^T = \begin{bmatrix} -2 & -3 & -1 \\ -2 & 0 & 2 \\ -2 & 3 & -1 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{\text{Adj } A}{|A|} = \frac{-1}{6}\begin{bmatrix} -2 & -3 & -1 \\ -2 & 0 & 2 \\ -2 & 3 & -1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & \frac{1}{2} & \frac{1}{6} \\ \frac{1}{3} & 0 & \frac{-1}{3} \\ \frac{1}{3} & \frac{-1}{2} & \frac{1}{6} \end{bmatrix}$$

**OR**

(iii) (b) : We have, $AA' = I$

$\Rightarrow \quad AA' = AA^{-1} \Rightarrow A^{-1} = A'$

$$\therefore \quad A^{-1} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -2 \\ 1 & -1 & 1 \end{bmatrix}$$

**38.** (i) We have,

$$AM = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 10+6 & 14+9 \\ 5+8 & 7+12 \end{bmatrix} = \begin{bmatrix} 16 & 23 \\ 13 & 19 \end{bmatrix}$$

(ii) $MA = \begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 10+7 & 15+28 \\ 4+3 & 6+12 \end{bmatrix} = \begin{bmatrix} 17 & 43 \\ 7 & 18 \end{bmatrix}$

(iii) (a) Given, $C - (A + M) = A$

$\Rightarrow \quad C = A + (A + M) \Rightarrow C = 2A + M$

$$\Rightarrow \quad C = 2\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} + \begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 2 & 8 \end{bmatrix} + \begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 9 & 13 \\ 4 & 11 \end{bmatrix}$$

**OR**

(iii) (b) Given, $D = AM - A$

$$\Rightarrow \quad \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 16 & 23 \\ 13 & 19 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 14 & 20 \\ 12 & 15 \end{bmatrix}$$

$\Rightarrow \quad a = 14, b = 20, c = 12, d = 15$

$\therefore \quad a + b + c + d = 14 + 20 + 12 + 15 = 61$

Class XII

# Monthly test drive

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

## Series-2 Inverse Trigonometric Functions

**Total Marks :** 80 **Time taken :** 60 Min.

### Only One Option Correct Type

**1.** The number of values of $x$ such that $\tan^{-1}x + \tan^{-1} 2x + \tan^{-1} 3x = \pi$ is

(a) 0 (b) 1 (c) 2 (d) $> 2$

**2.** If $\sin^{-1} x + \sin^{-1} 2x = \frac{\pi}{3}$, then $x$ is equal to

(a) $\frac{1}{\sqrt{7}}$ (b) $-\frac{1}{2\sqrt{5}}$ (c) $-\sqrt{\frac{3}{28}}$ (d) $\sqrt{\frac{3}{28}}$

**3.** If $\sin^{-1} x+\sin^{-1}y+\sin^{-1} z=\pi$, then $x^4+y^4+z^4+4x^2y^2z^2 = \lambda\,(y^2z^2 + z^2x^2 + x^2y^2)$ where $\lambda$ is equal to

(a) 1 (b) $-1$ (c) 2 (d) $-2$

**4.** If $x_1, x_2, x_3, x_4$ are the roots of $x^4 - x^3 \sin 2\alpha + x^2 \cos 2\alpha - x\cos\alpha - \sin\alpha = 0$, then $\sum_{i=1}^{4} \tan^{-1} x_i$ is equal to

(a) $\alpha$ (b) $\frac{\pi}{2} - \alpha$ (c) $-\alpha$ (d) $\pi - \alpha$

**5.** $\sin^{-1}\frac{1}{\sqrt{2}}+\sin^{-1}\left(\frac{\sqrt{2}-\sqrt{1}}{\sqrt{6}}\right)+\ldots$

$+\sin^{-1}\left(\frac{\sqrt{n}-\sqrt{n-1}}{\sqrt{n(n+1)}}\right)+\ldots\infty$ equals

(a) 0 (b) $\pi/4$ (c) $\pi/2$ (d) $3\pi/4$

**6.** If $\sin^{-1}(\cos^{-1}x) < 1$ and $\cos^{-1}(\cos^{-1}x) < 1$, then $x \in$

(a) (sin 1, sin (sin 1))
(b) (cos (cos 1), cos(sin1))
(c) (cos(sin 1), cos(cos1))
(d) $\phi$

### One or More than One Option(s) Correct Type

**7.** If $\cos^{-1} x - \sin^{-1} x = y$, then

(a) $x = 1/2, y = \pi/6$ (b) $x = -1/2, y = 5\pi/6$

(c) $x=\sqrt{3}/2, y=-\pi/6$ (d) $x=-\sqrt{3}/2, y=7\pi/6$

**8.** $\cos^{-1}\sqrt{\frac{a-x}{a-b}} = \sin^{-1}\sqrt{\frac{x-b}{a-b}}$ is possible if

(a) $a > x > b$
(b) $a < x < b$
(c) $a = x = b$
(d) $a > b$ and $x$, takes any value

**9.** Integral solution of the equation $\tan^{-1} x + \tan^{-1}(1/y) = \tan^{-1} 3$ is

(a) (2, 7) (b) (4, $-$ 13)
(c) (5, $-$ 8) (d) (1, 2)

**10.** Let $f(x)=e^{\cos^{-1}\sin\left(x+\frac{\pi}{3}\right)}$, then

(a) $f\left(\frac{8\pi}{9}\right)=e^{\frac{5\pi}{18}}$ (b) $f\left(\frac{8\pi}{9}\right)=e^{\frac{13\pi}{18}}$

(c) $f\left(-\frac{7\pi}{4}\right)=e^{\frac{\pi}{12}}$ (d) $f\left(-\frac{7\pi}{4}\right)=e^{\frac{11\pi}{12}}$

**11.** The value of $\tan\left[\frac{1}{2}\sin^{-1}\frac{2x}{1+x^2}+\frac{1}{2}\cos^{-1}\frac{1-x^2}{1+x^2}\right]$ is

(a) $\frac{2x}{1-x^2}$ if $0 \le x \le 1$ (b) $\frac{2x}{1-x^2}$ if $x < 1$

(c) not finite if $x > 1$ (d) none of these

**12.** If $\cos^{-1}x + (\sin^{-1}y)^2 = \frac{p\pi^2}{4}$ and $(\cos^{-1}x)(\sin^{-1}y)^2 = \frac{\pi^2}{16}$, then

(a) $0 \le p \le \frac{4}{\pi} + 1$ (b) $p \ge \frac{2}{\pi}$

(c) $p = 0, 1, 2$ (Integral values)

(d) none of these

**13.** Sum of series

$$\sum_{r=1}^{n} \sin^{-1}\left[\frac{2r+1}{r(r+1)\left(\sqrt{r^2+2r}+\sqrt{r^2-1}\right)}\right] \text{ is}$$

(a) $\frac{\pi}{2} - \sin^{-1}\left(\frac{1}{n+1}\right)$ (b) $\cos^{-1}\left(\frac{1}{n+1}\right)$

(c) $\cos^{-1}\left(\frac{1}{n+2}\right)$ (d) none of these

## Comprehension Type

### Paragraph for Q. No. 14 and 15

For $x \in \left(0, \frac{\pi}{4}\right)$, let $S_n = \sum_{r=1}^{2n} \sin(\sin^{-1} x^{3r-2})$,

$C_n = \sum_{r=1}^{2n} \cos(\cos^{-1} x^{3r-1})$ and $T_n = \sum_{r=1}^{2n} \tan(\tan^{-1} x^{3r})$,

where $n \in N$ and $n \ge 3$.

**14.** The correct order of $S_n$, $C_n$ and $T_n$ is given by

(a) $S_n > T_n > C_n$ (b) $S_n < C_n < T_n$

(c) $S_n < T_n < C_n$ (d) $S_n > C_n > T_n$

**15.** The value of $\lim_{n\to\infty}(S_n + C_n + T_n)$ is equal to

(a) $\frac{1}{1-x}$ (b) $\frac{x}{1-x}$

(c) $\frac{1}{1+x}$ (d) $\frac{x}{1+x}$

## Matrix Match Type

**16.** Match the following:

| Column-I | | Column-II | |
|---|---|---|---|
| (P) | If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$, then $x^2 + y^2 + z^2 + 2xyz =$ | (1) | 10 |
| (Q) | If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$ and $f(1) = 1, f(p+q) = f(p)\cdot f(q)$ $\forall p, q \in R$, then $x^{f(1)} + y^{f(2)} + z^{f(3)} - \frac{x+y+z}{x^{f(1)} + y^{f(2)} + z^{f(3)}} =$ | (2) | $n$ |
| (R) | If $\sum_{i=1}^{10} \sin^{-1} x_i = 5\pi$, then $\sum_{i=1}^{10} x_i^2 =$ | (3) | 2 |
| | | (4) | 1 |

(a) (P) → (3); (Q) → (1); (R) → (2)
(b) (P) → (4); (Q) → (3); (R) → (1)
(c) (P) → (2); (Q) → (1); (R) → (3)
(d) (P) → (4); (Q) → (3); (R) → (2)

## Numerical Answer Type

**17.** If $\cot^{-1}\left(\frac{n^2 - 10n + 21.6}{\pi}\right) > \frac{\pi}{6}, n \in N$, then the minimum value of $n$ is _____.

**18.** If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$, then

$\sum \frac{(x^{101} + y^{101})(x^{202} + y^{202})}{(x^{303} + y^{303})(x^{404} + y^{404})}$ is equal to _____.

**19.** If $K = \tan^{-1}\left(\frac{1}{2}\tan 2A\right) + \tan^{-1}(\cot A) + \tan^{-1}(\cot^3 A)$ and $\frac{\pi}{4} < A < \frac{\pi}{2}$, then $[K]$ equals _____.

(where [.] denotes greatest integer function)

**20.** The number of triplets satisfying $\sin^{-1}x + \cos^{-1}y + \sin^{-1}z = 2\pi$, is _____.

*Keys are published in this issue. Search now!* ☺

**SELF CHECK**

No. of questions attempted .....
No. of questions correct .....
Marks scored in percentage .....

**Check your score! If your score is**

| | | |
|---|---|---|
| > 90% | EXCELLENT WORK ! | You are well prepared to take the challenge of final exam. |
| 90-75% | GOOD WORK ! | You can score good in the final exam. |
| 74-60% | SATISFACTORY ! | You need to score more next time. |
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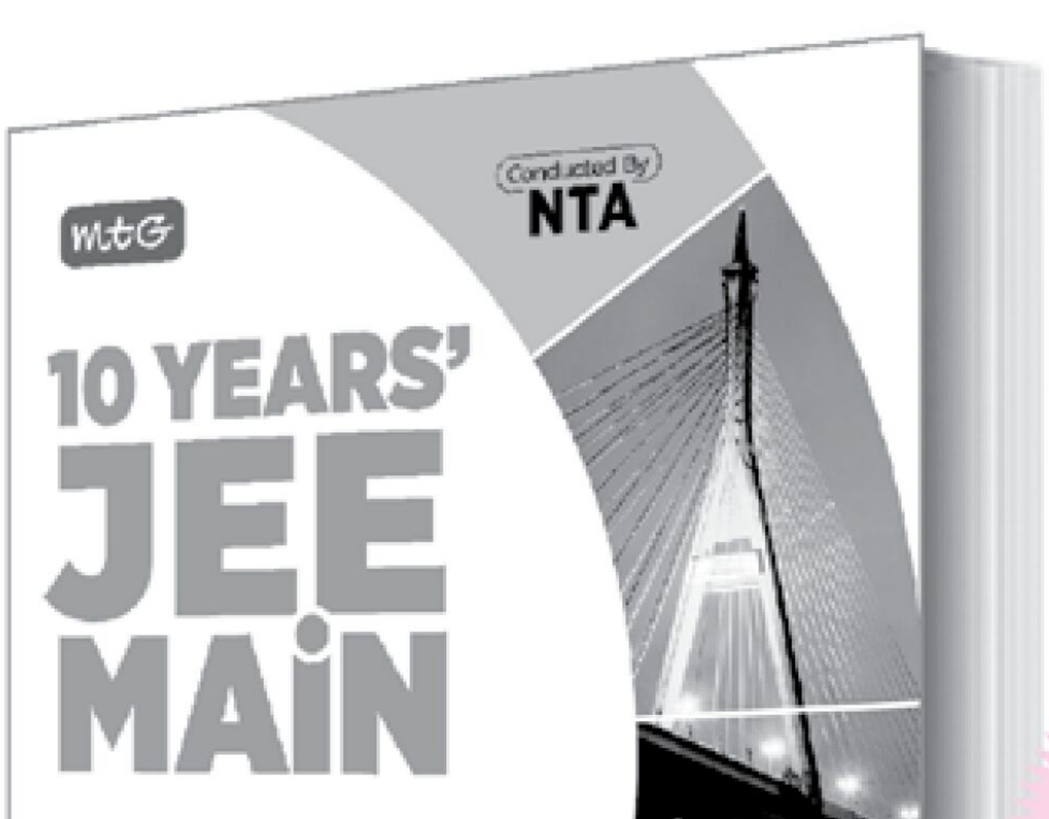


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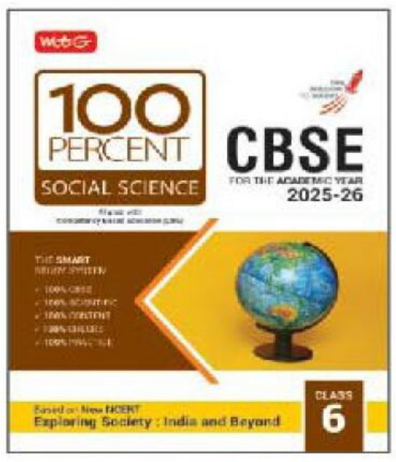


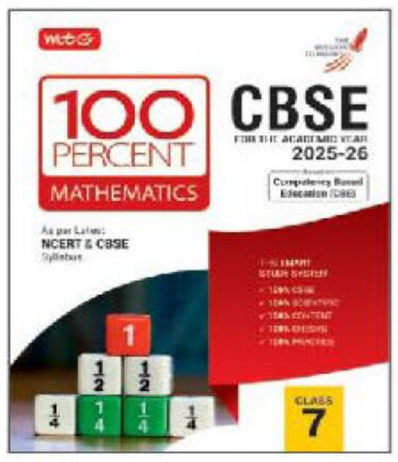

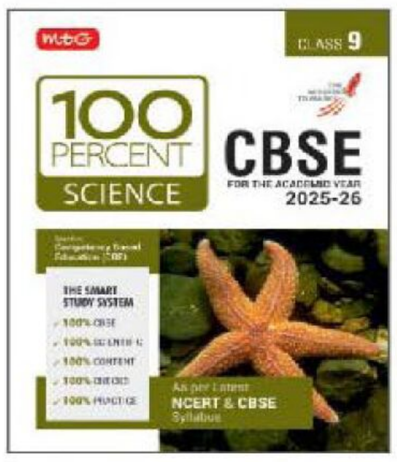


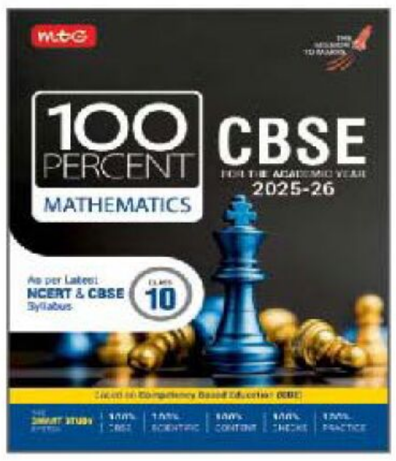


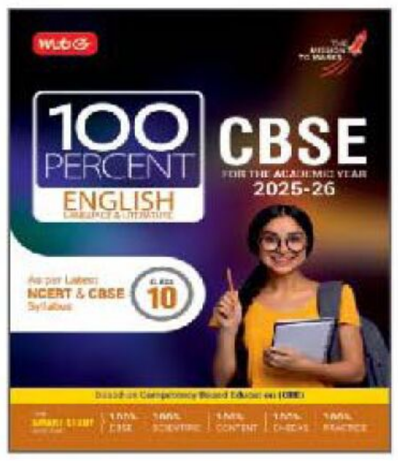


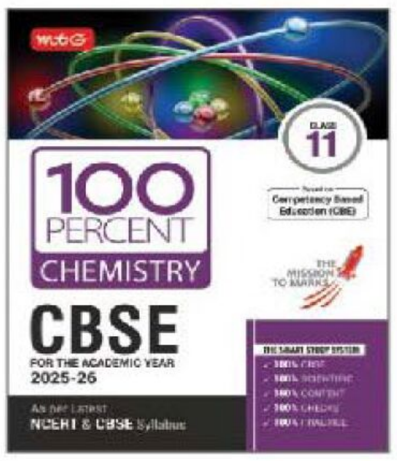


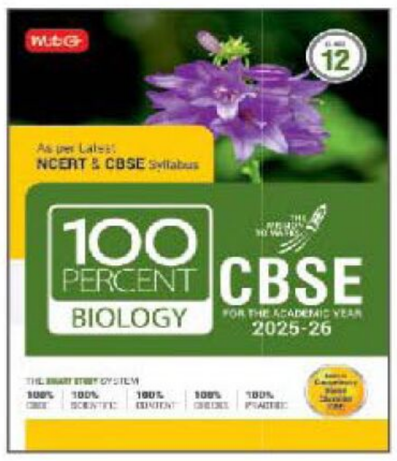


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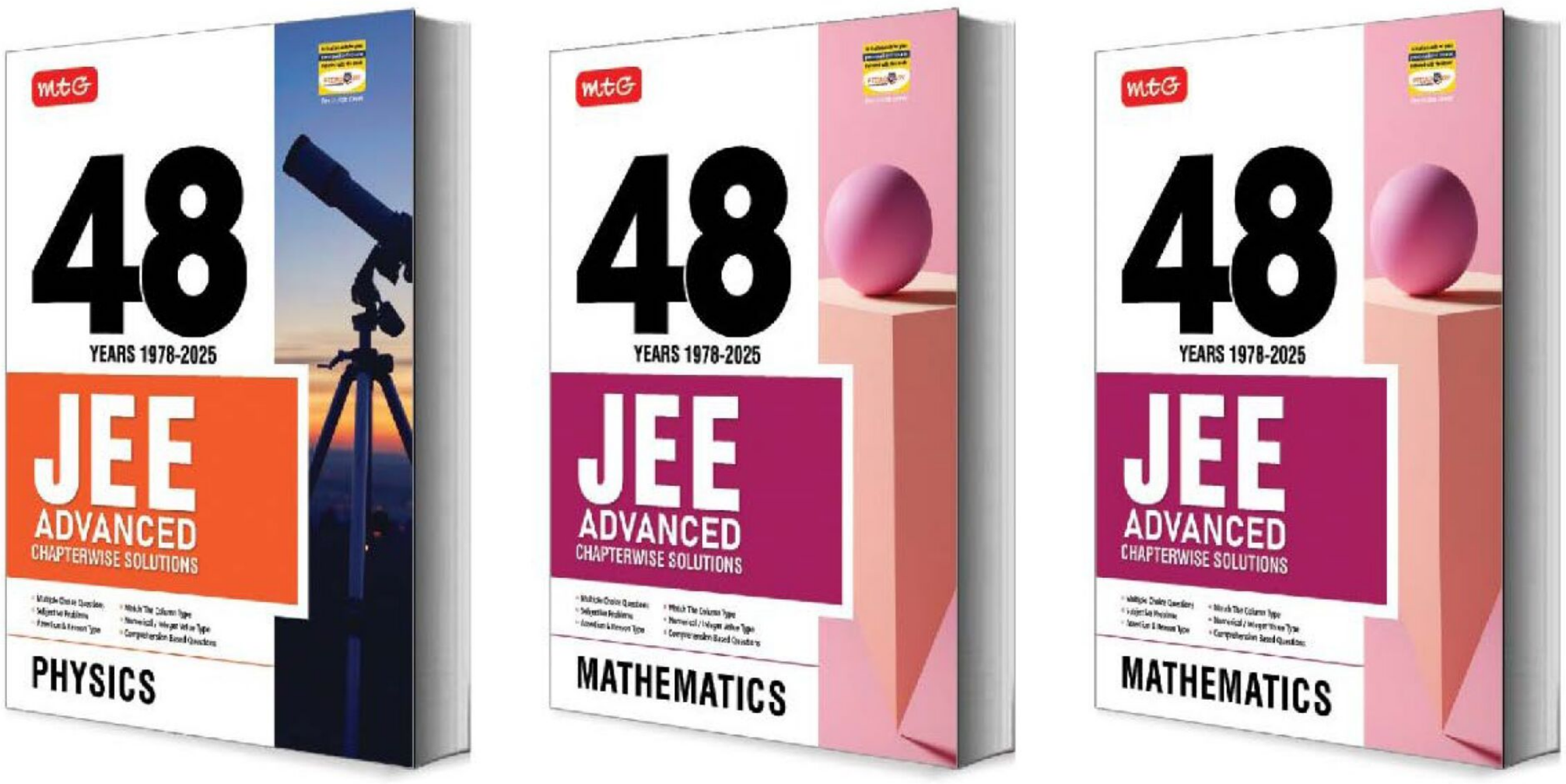


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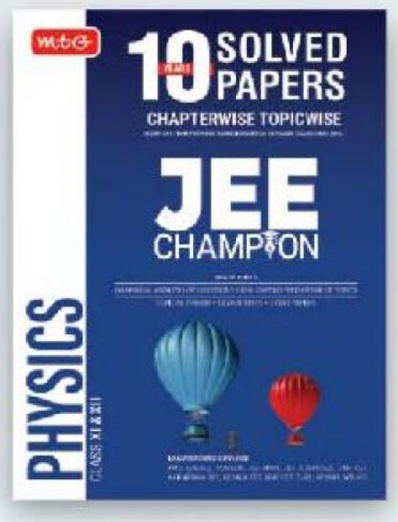


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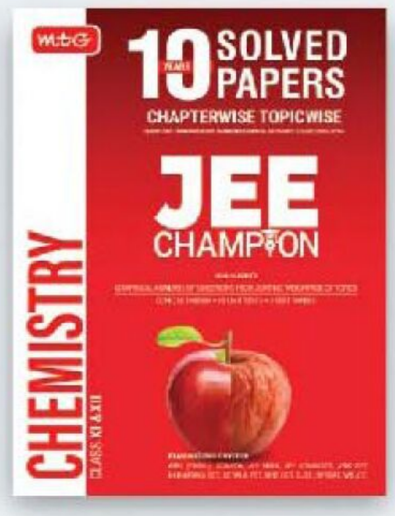


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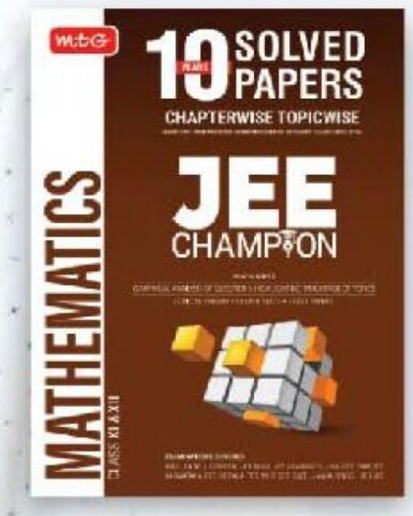


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